Problem B: Movement of an Object in a Microgravity Environment

1 The System

A probe nears an asteroid, lands upon that asteroid at time, and deploys a small rover on the asteroid's surface. The probe then departs from the asteroid, and the rover proceeds to explore the surface of the asteroid.

Exploration of the asteroid requires requires outlining the asteroid and the rover. Within the system, we assume that the asteroid is by far the most massive of the objects, and by that assumption we set the origin of the system to be the center of mass of the asteroid.

The surface of the asteroid is a surface S described by parametric function $\vec{r}$, where

$$S = \vec{r}(u,v) = \langle f_1(u,v), f_2(u,v), f_3(u,v)\rangle; (u,v) \in [0,2\pi] \times [0,\pi/2]$$

The gravitational potential field generated by the asteroid is given by the function

$$\vec{g}(\vec{r}) = -\frac{Gm_{ast}\hat{r}}{\|\vec{r}\|^2}$$

where $\vec{r}$ represents the vector from the asteroids center of mass-denoted $\hat{m}_{ast}$, to the object being acted upon by gravity, and $\hat{r}$ represents the unit vector in the direction of $\vec{r}$. So, for object of some mass m, the force of the gravity generated by the asteroid and acting upon it is $\vec{F}(\vec{r}) = m\vec{g}(\vec{r}))$

2 The Rover

The rover is a small circular disk of mass m_{rov} with a rounded side. It has two identical springs of spring constant k, used to propel it along the surface. A spring is located identically on each of its major sides such that either side may be landed upon without limiting the rovers mobility.

The rover is capable of compressing the spring a variable distance c, where $c \geq 0$ and $c \leq c_{max}$ where c_{max} represents the maximum compression distance of the spring. The rover is able to rotate the spring 2π radians upon the horizontal plane, and angle the spring up to $-\frac{\pi}{2}$ radians (perpendicular to the surface of the asteroid).

The rover moves by compressing and releasing the spring at the asteroid, and then by bouncing. Two variables for determining the final position of the rover are θ_{spring} and and ϕ_{spring}. θ_{spring} is the orientation of the spring with respect to the positive x-axis of the rover, which determines the direction travelled along the asteroid's surface. ϕ_{spring} is the angle of the spring with respect to the xz-axis of the rover, which affects the maximum distance reached from the center of mass of the asteroid.

θ_{spring} determines the direction the rover moves. The part of the asteroid that is traversed by the rover is determined by the value of θ_{spring}, so given any value θ_0 for θ_{spring}, the position of the rover on the asteroid after jumping can be considered as the position of the rover on the approximate ellipse defined by some function $g(\theta)$, $\theta \in [0,2\pi]$ formed by restricting the asteroid's surface in the direction of θ_0.

The movement of the rover along the direction defined by θ_0 is described by $\vec{p}(t)$ and is provided in polar coordinates.

$$\vec{p}(t) = \vec{p}_n + \vec{v}_n t + \vec{a}_n t^2 = \langle \vec{R}_n(t), \vec{A}_n(t) \rangle$$

Where $\vec{R}_n(t)$ represents the radial position of the rover at time t, and is the sum of the first elements of $\vec{p}_n$, $\vec{v}_n$, and $\vec{a}_n$, and $\vec{A}_n(t)$ represents the angular position of the rover at time t, and is the sum of the second elements.

$\vec{p}_n = \langle g(\theta_n), \theta_n \rangle$ and for all n $\vec{p}_n \in S$, $\vec{p}_n$ is either the starting position in the case of $n = 0$, or a point of contact and the start of a bounce.

$\vec{v}_n = \langle \sqrt{\frac{(1-L)^n (kc^2)}{m_{rov}}}, -\phi_n \rangle$, where L represents the percent loss of kinetic energy on collision, and $-\phi_n$ represents the angle of reflection off the surface S at $\vec{p}_n$, with $-\phi_0 = -\phi_{spring}$

$\vec{a}_n = \langle -\frac{m_{ast} G}{(\vec{R}_n(t))^2}, 0 \rangle$ where m_{ast} represents the mass of the asteroid, G represents the constant of gravitation, and $\vec{R}_n(t)$ represents the distance of the object from the asteroids center of mass (the origin). So, $\frac{d^2 R(t)}{dt^2} = m_{ast} G R(t)^{-2}$.

3 Assumptions and Constraints

We make certain restrictive assumptions in order to allow approximations regarding the asteroid. Firstly we assume that the asteroid is basically modelled by a bumpy ellipsoid and that the asteroid is of near uniform density. Secondly, we assume the asteroid is close enough to a sphere that we can approximate the gravity generated by the asteroid to be the gravity generated by a uniformly dense sphere, and thus a point mass at the center of mass of the asteroid.

We assume that from any point on the asteroid's surface, the asteroid is spherical enough with respect to any direction such that movement in the same direction upon the surface is closed. This is a limitation of the model.

Our model looks at relatively flat surfaces, and assumes that the rover is not hopping up inclined terrain, and would choose instead to hop over it. This is a limitation of the model.

We do not assume that the rover has a method of counteracting the bouncing in order to stop its movement, though such features could certainly be implemented. In the model we have provided, the quantity L-representing a percent energy loss per impact with the surface, is the sole method of stopping.

Without having a internal method to counteract bouncing, the rover as modelled is unlikely to be able to navigate steep and narrow terrain effectively.

4 Additional Issue 2

The size and shape of the asteroid impacts the predictions of the model by changing the value of L, and changing the path the rover would take in motion.

The size of the asteroid can be thought of in regards to mass and volume. L is dependent on the density of the asteroid at the surface, and if either the mass or the volume of the asteroid changes, then the density at the surface is likely to change. For higher densities, L is likely to be higher, and so we would expect the rover does not bounce as far for a given jump.

We make a fair amount of assumptions in regards to the shape of the asteroid. If the asteroid has a highly irregular surface, and is of such form as to be poorly approximated by a sphere, then our model is not likely to be useful in describing the position of the rover.