A mathematical model of the impacts of anti-aphrodisiacs on a population of *P. brassicae* and *T. brassicae*

Rosetta Roberts, Patience Lamb, Wesley Yockey

November 9, 2019

1 Introduction

The large cabbage white butterfly *Pieris brassicae* is known for its immense hunger and eating gardens around the world. Luckily, a natural counter for this species is the *Trichogramma brasssicae*, a wasp species which is known to practice brood parasitism by laying eggs next to *P. brassicae* eggs and having the larvae eat the eggs. The *T. brassicae* find the butterfly nests by following females covered in chemical compounds called anti-aphrodisiacs to their nests. Anti-aphrodisiacs are produced in order to mask or dissuade other males from mounting the female once shes covered in it, allowing for the female to not be bothered by many males and the male that placed the anti-aphrodisiac on her to be the most likely father [1]. We chose the Common Starling (*Sturnus vulgaris*) as a natural predator for *P. brassicae* and *T. brassicae*. The Common Starling has multiple food sources, not just insects. According to Audubon, the Common Starling's lifespan is $2-3$ years on average and have an adult survival fraction of 0.6. The birds reach sexual maturity at 1 year and flock in 20 birds per flock. [2]

2 Requirements

To determine the effects of anti-aphrodisiacs on the population of *P. brassicae* and *T. brassicae*, a mathematical model was developed to predict the population in the long-run and the best balance for the system.

3 Methodology

Research was done on the lifespan of *P. brassicae* and *T. brassicae* to include the interactions between the two species. It was discovered that *P. brassicae* have a lifespan of 32-64 days and lay eggs in clutches of 20-50, six to seven times in the course of eight days at the ages of 26.60-28.03 days. The eggs hatch two to seven hours after being laid [3]. *T. brassicae* on the other-hand parasitize 10 host eggs per day, the larvae reach sexual maturity 7-10 days later, living 7-14 days after hatching [4].

Following a parasitoid-host model [5], the team was able to deduce a differential equation for the relations of *P. brassicae* and *T. brassicae* over time. The differential equation was found to be

$$\frac{dL(t)}{dt} = \gamma A(t) - M(t) - \alpha P(t)L(t) \tag{1}$$

for the uninfected *P. brassicae* larvae density where $A(t)$ is the *P. brassicae* adult density, $M(t)$ is the rate of maturity of *P. brassicae* and $P(t)$ is the *T. brassicae* adult density. Multipliers of α and γ represent the rate of parasitism by *T. brassicae* and rate of larvae production of *P. brassicae* respectively. $M(t)$, the rate of maturity of *P. brassicae* was found via an exponential equation previously established in the parasitoid-host model as

$$M(t) = \gamma A(t - \tau_A) e^{-\int_{t-\tau_A}^{t} \alpha P(y)dy} \tag{2}$$

where τ_A is the maturation delay for *P. brassicae.*

The change in adult *P. brassicae* density over time is defined as

$$\frac{dA(t)}{dt} = M(t) - d_A A(t) - \alpha_B B \tag{3}$$

where d_A is defined as the death rate for the adult,α_B is the attack rate of the birds and B is the bird population *P. brassicae.*

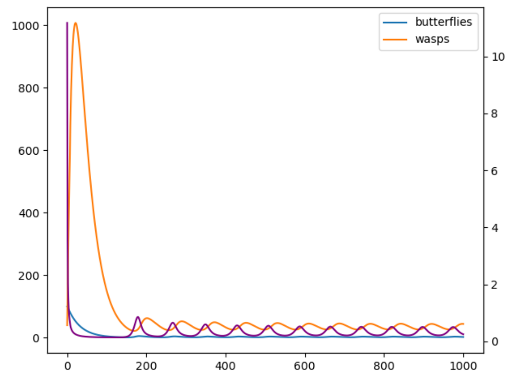
$\frac{dP(t)}{dt}$, the change of *T. brassicae* over time is established as

$$\frac{dP(t)}{dt} = \alpha P(t - \tau_p) - d_p P(t) - \alpha_B B \tag{4}$$

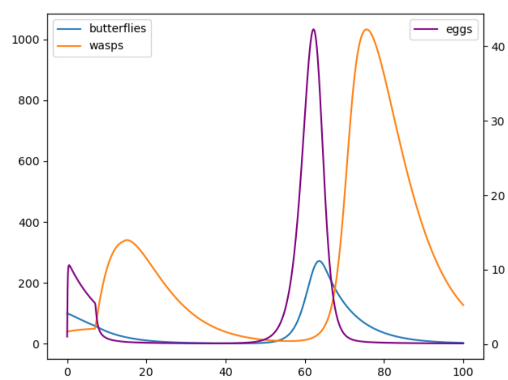
where τ_p is the maturation of *T. brassicae* and d_p is the death rate of *T. brassicae* The differential equations were entered into an initial value problem solver to solve for the respective populations of *P. brassicae* larvae, *P. brassicae* adults, and *T. brassicae* adults.

4 Analysis and Results

Varying the constants of α, γ, d_A, d_p, τ_A, τ_p, and the initial population densities resulted in some very unusual graphs. The final result was found via setting the value of α to 0.229, γ to 1.1, d_A to 0.03125, d_p to 0.0714, τ_A to 0.292, τ_p to 7, A_0 to 100, and P_0 to 40. Below is the graph with the set values.



Interestingly enough, the graph spiked in both populations originally and then created a periodic equilibrium in populations between *P. brassicae* and *T. brassicae* after a while. The number of butterflies stayed fairly consistent at a low number whereas the wasp number and the egg number fluctuated consistently. Both of the populations naturally reached a balance over the long period of time, allowing for neither of the populations to spike or go extinct. With the introduction of a bird, the population of both *P. brassicae* and *T. brassicae* become less periodic.



4.1 Important Assumptions

Assumptions include that the environmental factors such as food, water, and other predators are not important in determining population. The impact of the maturity and reproduction rates of the birds were negligible in relation to the *P. brassicae* and *T. brassicae* populations over the given time span.

References

[1] Martinus E. Huigens, Jozef B. Woelke, Foteini G. Pashalidou, T. Bukovinszky, Hans M. Smid, and Nina E. Fatouros. Chemical espionage on species-specific butterfly anti-aphrodisiacs by hitchhiking Trichogramma wasps. *Behavioral Ecology*, 21(3):470–478, 2010.

[2] Audubon Society. European starling, Oct 2019.

[3] CAB International. Pieris brassicae (large cabbage white).

[4] Bugological Control Systems. Trichogramma brassicae wasps.

[5] Ferdinand Pfab. Modeling parasitoid-host dynamics with delay differential equations.