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STUDENT VERSION

Falling Building Ice

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Abstract: We model the fall of a piece of ice which is falling from a high building in New York City.

STATEMENT

The Situation

In a *New York Times* article, [1], the author interviewed Frank Moscatelli, a clinical professor of physics at New York University, about the issues involved in ice falling from tall buildings in New York City.

"An object that falls from a building, such as a piece of ice, does not have to drop from tremendous heights to accelerate to dangerous speeds." Frank Moscatelli, a clinical professor of physics at New York University, said that ice could reach its maximum speed, between 60 and 70 miles per hour, from the top of a 15 - floor building.

"It would travel the same speed at that height as it would if it fell from the top of the city' s tallest building, 1 World Trade Center, whose spire climbs to 1,776 feet," Professor Moscatelli said. "That's because of terminal velocity, the maximum speed of a freely falling object."

"But whether that ice bounces off a pedestrian below or crushes him depends on the size of the ice. When the object hits you at, say, 64 m.p.h., it matters if it' s a penny or a brick," Professor Moscatelli said.

Let us examine the claims made by Professor Moscatelli.

1. Ice could reach its maximum speed, between 60 and 70 miles per hour, from the top of a 15-floor building.
2. Ice could attain the same maximum speed at that height as it would if it had fallen from the tip of the city's tallest building, 1 World Trade Center, whose spire climbs to 1,776 feet.

(Q1-2) Study Claims (1) and (2) above and ascertin whether or not they are valid.

And additionally, the professor claimed,

3. "When the object hits you at, say, 64 m.p.h., it matters if it's a penny or a brick."

Here are some additional questions we can ask ourselves.

(Q3) How much higher must the fall be to reach the actual terminal velocity? Is this possible?

(Q4) How much higher must the fall be to reach 99% or 99.99% of the actual TermVel = 29.057 m/s?

(Q5) What if we changed our computed value of a resistance term, say proportional to the velocity, but in the opposite direction to the velocity? After all, that was only a result of the professor's numbers he offered the reporter in conversation. So what if we had a (i) flat piece of ice (*parallel to the ground surface*) falling which would give rise (no pun intended) to more resistance and a bigger value of a? Or smaller value of a due to (ii) vertical sheet (*perpendicular to the ground*) of ice falling and thus meeting with less resistance.

(Q6) What will changes in the mass of the ice (assuming same shape and size so nature of resistance stays fixed) cause the terminal velocity? Any relationship between the two?

An interesting source is found in the paper, "The Implication of Energy Efficient Building Envelope Details for Ice and Snoe Formation Patterns on Buildings" [2].

Begin Modeling

We can use Newton' s Second Law of Motion, which says that the sum of the forces acting on an object is equal to its mass times its acceleration, A, so called $F = m \cdot A$. in this way we build a differential equation in which there are two forces acting on the mass of ice, (F1) force due to acceleration of gravity and ($F2$) force of resistance due to air resistance. Force $F1$ is equal to $m \cdot g$, where m is the mass in kilograms and g is the acceleration due to gravity 9.8 meters per second per second or 9.8 m/s^2. Force $F2$ is proportional to the velocity, say '$a \cdot h'(t)$ where here, $h(t)$ is the distance fallen at time t and the constant of proportionality, a, is a constant which depends upon the shape of the falling object and its interface with air, e.g., sheet of ice laying out flat or parallel to the ground would experience more resistance due to movement through the air than that same sheet if it were vertical and perpendicular to the ground.

So here is our force equation (1), which, as we can see, is also a differential equation! Here we designate $h(t)$ to be the height fallen in meters at time t in seconds, with initial condition $h(0) = 0$, at starting point of height 49.5 m, the converted to metric system height of the 15 story building. Now if we also presume the ice was traveling with a velocity of 0 m/s as it begins its fall (reasonable!) we have a second initial condition, $h'(0) = 0$.

So let us see exactly what our Initial Value Problem looks like with one second order differential equation in $h(t)$ and two initial conditions. Here negative numbers indicate upward motion and positive numbers indicate downward motion, the distance elapsed in the fall. We use $a > 0$ as our constant of proportionality due to air resistance.

Go ahead and confirm the differential equation model in (1). Then use it to address the issues and questions raised above.

$$m \cdot h''(t) = F1 - F2 = m \cdot g - a \cdot h'(t) \tag{1}$$

where $F1 = m \cdot g$ and $F2 = a \cdot h'(t)$.

REFERENCES

[1] Haag, Matthew. 23 December 2019. Add Falling Ice From Glass Towers to a New Yorker's List of Worries. *The New York Times*. 23 December 2019. Accessed 24 December 2019.

[2] Norris N., D. André, P. Adams, M. Carter, and R. Stangl. 2014. The Implication of Energy Efficient Building Envelope Details for Ice and Snoe Formation Patterns on Buildings. *Proceedings of the 14th Canadian Conference on Building Science and Technology*. pp.63-75.