www.simiode.org

SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

Reasons for SIMIODE

The COMAP/SIAM report, *GAIMME - Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME 2016) encourages us to do modeling throughout the mathematics curriculum and shows us how to do this in rich detail, including the tricky part of assessment.

The *2015 CUPM Curriculum Guide* (CUPM 2015) recommends,

> "There are major applications involving differential equations in all areas of science and engineering, and so many of these should be included in the ODE course to show students the relevance and importance of this topic."

The report also does a reality check by admitting that many topics in traditional differential equations course have been de-emphasized while use of modeling and technology has increased.

We propose a serious paradigm shift to a modeling-based approach for teaching differential equations. To that end we are building a community of support, SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations, for teachers and students at www.simiode.org.

SIMIODE is about teaching differential equations using modeling and technology upfront and throughout the learning process. We offer a community, in which colleagues can explore, communicate, collaborate, publish, teach, contribute, archive, etc. Access to this community is free. Teachers who register can gain access to double-blind, peer-reviewed materials with collegial comments on pedagogy and use in class.

The modeling-first approach closely parallels the underlying principles found in problem-based learning, inductive learning, and inquiry-based learning, which suggest that students learn best by doing and retain best when they construct their own paradigms. See Prince &Felder 2006, Cotic & Zuljan 2009, Brunning EtAl. 2004, and Sanders 2009. Additionally, putting the mathematics into the context of real world problems, makes the subject meaningful, applicable, interesting, and powerful in the eyes of the students. This can aid with student attitudes about mathematics, resulting in increased curiosity, persistence, and perceived usefulness according to research by Silvia 2008 and Rasmussen & Kwon 2007. Moreover, such an approach can enhance transferability of the mathematical knowledge as it is based in a reality with vivid imagery.

In a seminal study on inductive teaching and learning in the premier engineering education journal, the American Society for Engineering Education's (ASEE) *Journal of Engineering Education*, the authors conclude that ". . . inductive methods are consistently found to be at least equal to, and in general more effective than, traditional deductive methods for achieving a broad range of learning outcomes." Prince & Felder [page 1] 2006.

One of the authors, Prince [page 55] 2007, in a closing essay for *PRISM*, the magazine of ASEE,

> "Another well-entrenched tenet of traditional instruction is the notion that students must first master the underlying principles and theories of a discipline before being asked to solve substantive problems in that discipline.
>
> "An analysis of the literature as rendered in Prince & Felder (2006) suggests that there are sometimes good reason to `teach backwards' by introducing students to complex and realistic problems before exposing them to the relevant theory and equations."

Colleagues around the academic world believe in inductive methods that are in touch with the real world and are using this same approach. In a paper for teachers of American history, Lendol Calder forcefully supports an inductive approach (Calder 2007), by quoting the distinguished professor of history, Charles G. Sellers (UC Berkeley):

> "The notion that students must first be given facts and then at some distant time in the future will 'think' about them is both a cover-up and a perversion of pedagogy. . . . One does not collect facts he does not need, hang on to them, and then stumble across the propitious moments to use them. One is first perplexed by a problem and then makes use of the facts to achieve a solution."

We know modeling reality motivates and creates mathematics. It has been a *raison d'être* for much of mathematics and occurs naturally. We can see it happening! Give elementary school students a collection of buttons and ask them to describe what they see. They categorize and sort them, on color, size, shape, mass, etc. Then they invent, ON THE SPOT, the notion of histogram to render their narrative. This is what happens when modeling comes first; mathematics is created and used by students. So it is at the level of differential equations.

What Does SIMIODE Offer?

SIMIODE is a supporting Community of Practice at www.simiode.org with resources for teaching differential equations through modeling and real-world situations. SIMIODE advocates and supports an inductive approach to learning differential equations through context with the use of Modeling Scenarios.

Modeling Scenarios form the heart of the teaching materials found in SIMIODE in which a situation is presented which leads to model building using differential equations. Such scenarios often have data or the opportunity to collect data, in the latter case through the SIMIODE YouTube channel (SIMIODE 2016). These activities run the gamut from quick, in class activities, to long projects.

SIMIODE conducts paper sessions, minicourses, and workshops at national mathematics meetings in the United States for the American Mathematics Society and the Mathematics Association of America and offers on site faculty development workshops, e.g., Savannah State University, Savannah GA USA and Universidad Tecnológica de Panamá, Panama City PANAMA.

We list some of the modeling possibilities: dissipation of an intraocular gas bubble after retinal surgery, chemical kinetics of reactions, spread of an oil slick, feral cat control, tuned mass dampers to keep big buildings from swaying too much, LSD in the body, whiffle ball or shuttlecock falling, stadium design, pendulum motion, ascent rates in SCUBA diving, absorption of drugs like ibuprofen in the human body, and spread of the word JUMBO in literature. These can all be found in SIMIODE (SIMIODE 2016a) and are freely available. Each of these scenarios can motivate at least one (and often many more) differential equation concept.-They draw students into the study of these equations, because the content interests them and makes them curious. This is where the language and methods of the mathematics of differential equations stand ready to assist in discovery and exploration. Students are motivated to seek out and learn the tools of differential equations in order to address the context of the situation that piques their interest.

Currently there are over two dozen FREE on-line, curated, differential equations text books to support the traditional basics and save students money – lots of money. These are referenced and reviewed in SIMIODE (Winkel 2015a). SIMIODE enriches the study of differential equations with motivational Modeling Scenarios which draw students to the mathematics. More important, SIMIODE is a community. Teachers can form project areas and interest groups to support inquiry and material development with colleagues around the world. Students can work together with each other. A whole class or section of a course can be created as a group by a teacher for whatever purpose the teacher desires. Small groups and project teams in a class or across the globe can collaborate on writing, data analysis, videos, etc. and keep their results and writing in their own project area in SIMIODE. SIMIODE has many rich aspects – community, learning and teaching, resources, and modeling.

Modeling Scenario illustrations

We present details on four Modeling Scenarios from SIMIODE to illustrate the details of a typical approach to teaching differential equations through modeling.

Population death and immigration modeling

The most used Modeling Scenario (Winkel 2015) of SIMIODE's many resources is a population model using m&ms as members of the population. On the first day of differential equations class the teacher says, "We are going to model death and immigration," and gives each student two cups, one paper plate, and a bag of m&ms with a sheet of instructions. Count out 50 m&ms, place in one cup, and toss onto the plate. If m side is up that m&m dies, remove from population, and once all dead are cleared out immigrate 10 m&ms back into the population.

Count and record the population. Keep doing this. "What happens?" the teacher asks after about 10 minutes. Discussion ensues, most importantly modeling begins, leading to the natural discovery of difference equation and differential equation mathematics and "inventing" concepts of iteration, change and rate of change, equilibrium value, long term behavior, parameter estimation, and solution strategies. What's more, students will refer to this first day experience throughout the course for its approach, content, terms, use of data to motivate the study, power to illustrate modeling, methods, and excitement (Yagodich 2016).

Figure 1. Resources and first generation simulation for m&ms data.

> "In my class evaluations, students were asked `What class assignment or activity did you find to be the most useful?' One student answered, `Believe it or not, the M&M activity on the first day of class really stood out to me. It helped show the real world applications of Differential Equations and I thought it was amazing that we could build an equation using real world data.' The fact that a student remembered this fairly short activity done the very first day of class all the way to the end of the semester made a big impact for me and planning for future semesters!"

Thousands of students have experienced this approach with great success. For a variation on this activity see (Winkel 2016) in which the immigration rate is a mystery number selected by each team of students and the resulting data is passed along to a different team to determine that immigration rate.

All this material is in the Student Version of the Modeling Scenario. There is also a Teacher Version in which the author offers details on uses, pitfalls, modifications, descriptions of what to do, etc. For example, in the Teacher Version of this Modeling Scenario there is material to help the teacher take students on the transition from a discrete differential equation model to a continuous differential equation model. Teacher Version materials are available only to teaching members of the community, which is freely offered through a registration process at https://simiode.org/register/ .

Torricelli's Law for a Falling Column of Water

In a self-contained Modeling Scenario (Winkel 2015b) students are presented with a mathematical formulation of Torricelli's Law for a Falling Column of Water. The goal is to build a differential equation model which describes the rate of change of the height of the column of water as the water exits through a small hole at the base of the column.

The material in the Modeling Scenario uses the Conservation of Energy Principle which says that the total energy (kinetic plus potential) of a drop of water in a column of water is conserved as it falls. This provides an expression for the exit velocity of the water from a small aperture in the container. Couple this with the cross sectional area of the aperture and one can model the rate at which the volume of water and hence the height of the constant cross-sectional column of water falls. With this principle, the very commonly used approach in modeling in which some quantity is computed in two ways and both computations are equated is employed to form a differential equation model.

The resulting model for the change in volume of the column of water at time t is

$$A(h(t))\ h'(t) = -\ \alpha\ a\ (2\ g\ h(t))^{1/2}$$

where h(t) is the height of the column of water at time t; A(h(t)) is the cross sectional area of the column at time t; a is the constant cross sectional area of the exit hole; g is the acceleration due to gravity; and α is an empirical constant called the discharge or contraction coefficiens, which is an unknown parameter, indicating just what percentage of the water that could exit through the exit hole actually does leave, the difference being due to the radius and friction at the exit orifice.

Students can collect data from any one of a number of column and exit hole configuration videos at the SIMIODE YouTube channel (SIMIODE 2016) with information on the constant cross sectional area of the column, A = A(h(t)), and the exit hole, a. Students can stop the video, observe the time on the digital clock, and estimate the height of the column of water, from the images presented on the screen.

Sublimation data

A simple experiment in which a small block of dry ice (frozen carbon dioxide) sublimates can prove to be a very interesting modeling scenario (Winkel 2015c) and serves as an introduction and reason for studying separation of variables techniques. A schematic of equipment needed to collect data is shown in Figure 2.

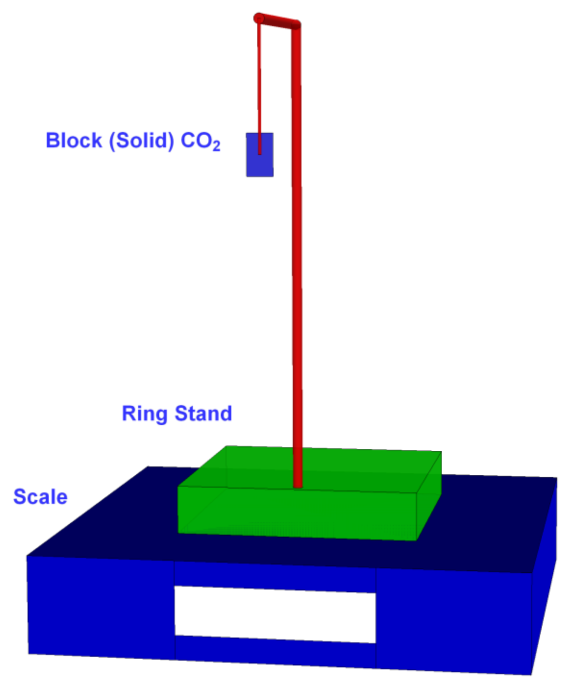

Figure 2. Sketch of apparatus for collecting sublimation of dry ice data.

Warning: The first time we collected data we had placed the dry ice on the metal plate of a scientific lab scale and the mass went up as time progressed! Water was freezing and condensing on the plate faster than the dry ice was sublimating. We recommend that, with this and all experiments, one performs a “dry run.”

Students can build a model of the rate at which the mass changes in terms of the geometry (in our case a cube of dry ice) and they will conjecture a differential equation model with initial conditions for the data of the sort:

$$m'(t) = -k\,m(t)^r, \quad m(0) = 7.57$$

Upon solving this differential equation and fitting the solution to the data, a best fit occurs when r = 0.70 which is close to what one might expect with r = 2/3. The latter is due to the fact that the mass is proportional to the volume and so the rate of decrease in mass or volume is proportional to surface area, i.e. volume (or mass) raised to the 2/3 power. A plot of the data and the model is very convincing (Figure 3).

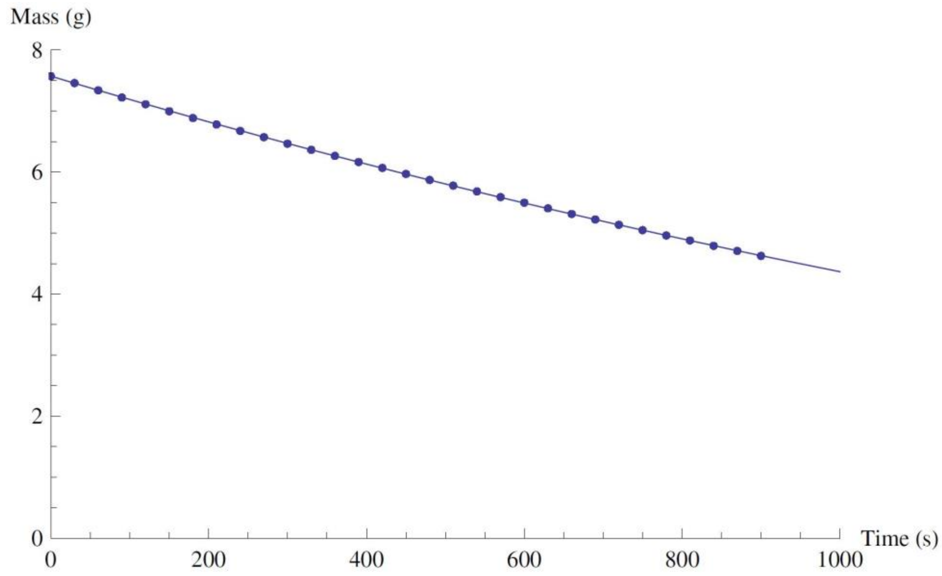

Figure 3. Data and model fitted to the data for sublimation of dry ice.

Epidemic in a Boarding School

Nonlinear differential equations are useful tools when modeling certain phenomena. We consider the spread of flu in an English boarding school (Miller 2015) modeling scenario in there are Susceptibles, Infectives, and Removed members of the population. Students readily build models from simplifying assumptions and diagrams.

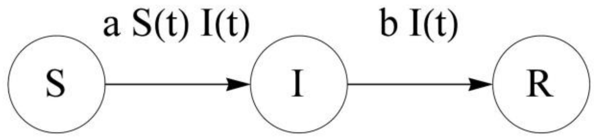

Figure 4. Schematic student develop for setting up epidemic model**.**

It is quite easy for students to build the nonlinear difference equations.

$$
\begin{aligned}
S_{n+1} &= S_n - aS_nI_n \\
I_{n+1} &= I_n + aS_nI_n - bI_n \\
R_{n+1} &= R_n + bI_n.
\end{aligned}
$$

Or they can build a differential equations model,

$$
\begin{aligned}
S'(t) &= -aS(t)I(t) \\
I'(t) &= aS(t)I(t) - bI(t) \\
R'(t) &= bI(t).
\end{aligned}
$$

Then using Euler's Method with small step sizes in a spreadsheet students can estimate the parameters a and b using graphical feedback or Excel's Solver Command to minimize the sum of square errors between the model solution and the data and obtain plots of the model over the data. We see the results, first in the discrete case using Excel (Figure 4) and then in the continuous case, using Mathematica, for example (Figure 5).

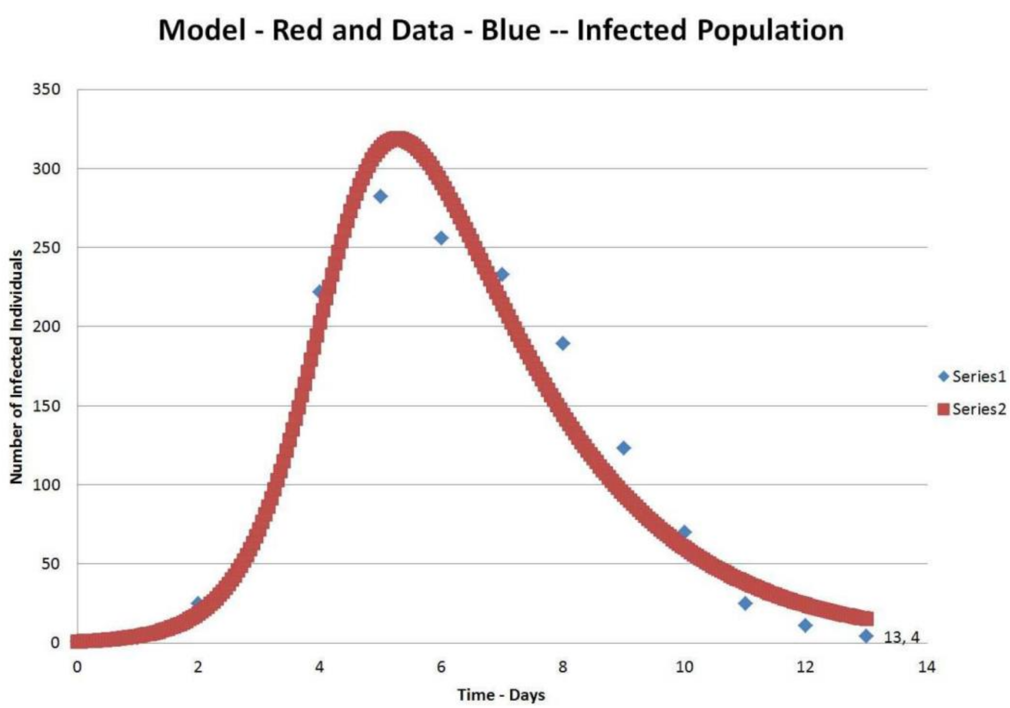

Figure 4. Parameter estimation result for epidemic model using discrete model and Excel spreadsheet.

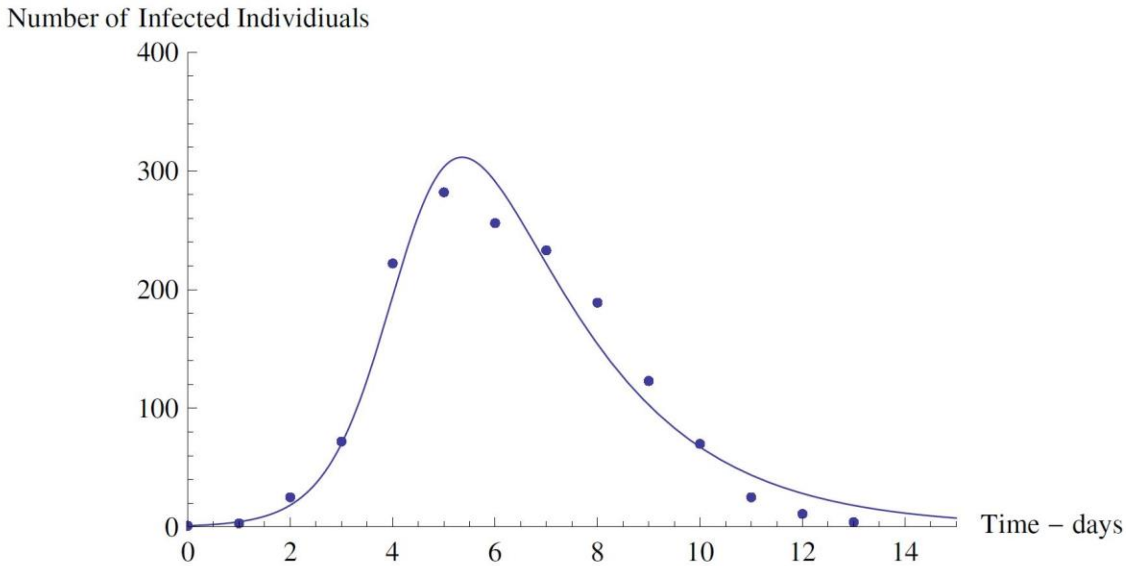

Figure 5. Parameter estimation result for epidemic model using continuous model and Mathematica.

Conclusion

We have supported the notion that students learn mathematics best when it is in context and there is interest and curiosity present. We have presented details on a community of teachers and learners in SIMIODE who are interested in teaching and learning differential equations through modeling of realistic situations. Finally, we have given four examples of Modeling Scenario material available at SIMIODE. We invite readers to join our community at www.simiode.org .

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Brian Winkel, Director SIMIODE, 12 March 2020