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STUDENT VERSION

Styrofoam Ball Fall

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STATEMENT

We are given data (see Table 1) on a falling Styrofoam ball and we seek to model this motion.

Time - s	Distance - m
0	0
0.132	0.075
0.232	0.26
0.332	0.525
0.432	0.87
0.532	1.27
0.632	1.73
0.732	2.23
0.832	2.77
0.932	3.35

Table 1. Data from [1] on falling Styrofoam ball.

We propose two models for $y(t)$ the distance fallen by the Styrofoam ball in meters at time t in seconds in which positive distances and velocities are downward.

1. The ball meets with resistance due to the air which is proportional to velocity, $y'(t)$. We model this with $-ay'(t)$ where we presume a is positive.
2. The ball meets with resistance due to the air which is proportional to velocity squared, $y'(t)^2$. We model this with $-ay'(t)^2$ where we presume a is positive.

In both cases (1) and (2) we seek to determine the parameter a such that the model fits the data best in the sense of minimizing the sum of square errors between the observed data and the model over all data points.

Activitiy 1: For both cases (1) and (2) build a model using Newton's Second Law of Motion (1) which says that the mass times the acceleration of that mass is equal to the sum of all the forces acting on the mass. In our case this means

$$m \cdot y''(t) = \text{sum of forces acting on the mass } m\,. \tag{1}$$

Activitiy 2: For both cases (1) and (2) solve your differential equation.

Activitiy 3: Estimate any parameters based on the data. Use the method in which you minimize the sum of the differences squared between the data and the model at each data value in terms of the parameter(s). Hint: You can assume the acceleration on the mass due to gravity is $g = 9.8m/s^2$.

Activitiy 4: Compare your models' prediction using a plot of the model AND the data. Examine the plot of the residiuals, i.e. differences between data and model predictions. This plot should look random with no patterns. Which of the two models would saiy is a better one?

Activitiy 5: We offer a way of determining which model is best. It is called the Akaike Information Criterion (AIC). A readable reference to AIC can be found in [2, p. 138].

The Akaike Information Criterion (2), where k is number of parameters to fit, n is the number of observations, and SSE is the minimum residual sum of square errors) (or just sum of square errors SSE) is

$$AIC = 2(1+k) + n \log \left[\frac{SSE}{n} \right]\,. \tag{2}$$

NB: When SSE is small then the log function could be negative.

Smaller (even negative) values of AIC represent a higher level of statistical support for the corresponding model. A difference of 2 in the AIC's is needed to suggest a significant difference.

The smallest AIC means the best model - in terms of balancing ACCURACY (SSE) and COMPLEXITY (k).

So after we complete the model analyses (1) - (2) we will see if one of the models is best.

Use the parameter(s) and sum of square errors in terms of the parameter(s) in Model (1) - resistance proportional to velocity and Model (2) - resistance proportional to velocity squared in AIC to determine which model is best and hence which is more appropriate description of resistance to the fall of the Styrofoam ball.

REFERENCES

[1] Greenwood, M. S., C. Hanna, and J. Milton. 1986. Air Resistance Acting on a Sphere. *The Physics Teacher*. 24: 153-159.

[2] Ledder, G. 2013. *Mathematics for the Life Sciences*. New York: Springer.