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TEACHER VERSION
Corona Virus Aerosols

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Abstract: Students will use reported data [2, 3] on the motion of the corona virus aerosols to model just how far they can go from the mouth of a coughing or sneezing infected person and compare that to the recommended social distancing distance of 6 feet which is the suggested distance for mitigating the spread of the virus.

Keywords: corona virus, projectile motion, resistance, free fall, social distancing

Tags: linear system, model, projectile motion, resistance

STATEMENT

As of 11 March 2020 the level of COVID-19 (coronoa virus) found in populations around the world led the World Heath Organization [4] to call the situation a pandemic. There were a number of suggestions to help stop the spread of the virus, among them washing hands regularly and thoroughly and social distancing. The latter refers to restricting assemblies of people to 10 or less and in some cases in self-home quarantine. It has also been suggested to keep one's distance (social distancing) to no closer than 6 feet in any social engagements. Media have been promoting this 6 feet distance, for example [1].

> "If you do let your kids outside to play with others, make sure the children keep at least 6 feet of distance from other children (which can be very hard for younger children to abide by). That's because the virus can be transmitted between people who are in close contact with each other – about a 6-foot radius."

We consider two pieces of information on which we ask you to build a model of the trajectory of an aeresol of the coronoa virus (the way the virus is transmitted through the air) to examine this 6-foot criteria.

Linsey Marr is at https://www.cee.vt.edu/profile/?pid=lmarr and her organiation is at https://www.air.cee.vt.edu.

1. Dr. Linsey Marr[2], an expert in the transmission of viruses by aerosol at Virginia Tech in Blacksburg VA USA, says that an aerosol virus droplet dropped from 6 feet takes 34 minutes to reach the ground. The virus does not linger in the air at high enough levels to be a risk to most people who are not physically near an infected person. The virus, "... can travel through the air and stay suspended for a period of about a half-hour." Further, "Dr. Marr said based on physics, an aerosol released at a height of about six feet would fall to the ground after 34 minutes. ... Dr. Marr compared this to cigarette smoke or a foggy breath on a frosty day. The closer and sooner another person is to the exhaled smoke or breath, the more of a whiff they might catch; for anyone farther than a few feet away, there is too little of the virus in the air to be any danger."
2. From [3] we quote from the Abstract, highlighting the information we seek in BoldFace on the initial speed of an aerosol from a cough for female and male.

   Abstract: Cough airflow dynamics have been previously studied using a variety of experimental methods. In this study, real - time, non - invasive shadowgraph imaging was applied to obtain additional analyses of cough air flows produced by healthy volunteers. Twenty healthy volunteers (10 women, mean age 32.2 ± 12.9 years; 10 men, mean age 25.3 ± 2.5 years) were asked to cough freely, then into their sleeves (as per current US CDC recommendations) in this study to analyze cough airflow dynamics.

   For the 10 females (cases 1–10), their maximum detectable cough propagation distances ranged from 0.16–.55 m, with **maximum derived velocities of 2.2–5.0 m/s**, and their maximum detectable 2 - D projected areas ranged from 0.010–0.11 m^2, with maximum derived expansion rates of 0.15–0.55 m^2/s.

   For the 10 males (cases 11–20), their maximum detectable cough propagation distances ranged from 0.31–0.64 m, with **maximum derived velocities of 3.2–14 m/s**, and their maximum detectable 2 - D projected areas ranged from 0.04–0.14 m^2, with maximum derived expansion rates of 0.25–1.4 m^2/s.][3]

   We could presume the maximum value of 14 m/s or 45.9318 ft/s or, in our case (45.9318 ft/s) · (60 s/min) = 2755.91 ft/min and "reasonable" value of, say, 8 m/s or 1574.8 ft/min. Although, this is significantly higher than the highest value for females, we could err on the side of excess. But also consider a reasonable value for the ejection velocity from the mouth of an infected person, say, of 8 m/s or 1574.8 ft/min.

So let us tackle building a model for the projectile path of a corona virus aerosol which is ejected through the cough or sneeze from the mouth of an infected person whose mouth is at a height of 6 feet above the floor.

Consider the following steps:

1. we need to know the initial speed of an aerosol from a cough and then with this information

2. we use the free falling information that "it takes 30 minutes for the virus to fall from the height of 6 ft"[2] to determine a model for the path of the aerosol.

In particular we consider the resistance due to the air in the context of a projectile motion problem to see the path of the aerosol from the cough origin to determine just how far away from the infected person's mouth the aerosol can really go. We will attack the steps in reverse order, namely, Step (2) and then using the results turn to Step (1).

Modeling Free Fall of Virus Aerosol - Step 2

Use Newton's Second Law of motion which says that the mass (our small aerosol is our mass in this situation) times the acceleration on that mass (m) is equal to the sum of the external forces acting on the mass. Here, we have two forces to consider as acting on the aerosol mass (a) $m \cdot g$ where g is the acceleration due to gravity and (b) the resistance term due to air, namely, $a \cdot v(t)$, where a is a proportionality constant and $v(t)$ is the velocity of the mass aerosol at time t, say in minutes. Our distances will be in feet.

Then use the information from Dr. Marr offered above about the time for free fall of a virus aerosol to determine a full model for motion of the aerosol through air with resistance.

There are several variations of resistance and we list them here for your consideration to incorporate into your model.

1. Resistance due to the medium is proporational to the velocity.
2. Resistance due to the medium is proporational to the velocity squared.
3. Resistance due to the medium is proporational to the velocity raised to some unknown power, say r.

Consider each of these cases and construct separate models and estimate parameters based on Dr. Marr's information.

Modeling Projectile Motion of Virus Aerosol - Step 1

Now consider the information from Cough Airflow Dynamics to build a complete model of the projectile motion of virus aerosol after a cough and determine just how far the aerosol carrying the virus can reach.

Do this for all three cases above, (1) resistance due to the medium is proporational to the velocity, (2) resistance due to the medium is proporational to the velocity squared, and (3) resistance due to the medium is proporational to the velocity raised to some unknown power, say r.

Finally, compare these case results to the advised social distancing suggestions of 6 feet and if there is a difference discuss what could be at fault.

Conclusion and Reflection

Discuss how your results compare with the public guidance of maintaining a social distance of more than 6 feet so the virus is not transmitted from an infected person who is infected to a susceptible person. What analysis can you find to support the 6 feet number and what scrutiny to your model can you offer to be more sure of your results and recommendations about social distancing?

REFERENCES

[1] Godby, Maria. 2020. It's Time To Get Serious About Social Distancing. Here's How. National Public Radio (NPR). https://www.npr.org/sections/health-shots/2020/03/17/817251610/its-time-to-get-serious-about-social-distancing-here-s-how.

[2] Mandavilli, Apoorva. 2020. How Long Will Coronavirus Live on Surfaces or in the Air Around You? New York Times. 17 March 2020. https://www.nytimes.com/2020/03/17/health/coronavirus-surfaces-aerosols.html. Accessed 19 March 2020.

[3] Tang, Julian W., Andre Nicolle, Jovan Pantelic, Gerald C. Koh, Liang De Wang, Muhammad Amin, Christian A. Klettner, David K. W. Cheong, Chandra Sekhar, and Kwok Wai Tham. 2012. Airflow Dynamics of Coughing in Healthy Human Volunteers by Shadowgraph Imaging: An Aid to Aerosol Infection Control. *PLOS ONE* . 7(4): e34818. ttps://doi.org/10.1371/journal.pone.0034818. Accessed 18 March 2020.

[4] World Heatlh Organization. 2020. WHO Director-General's opening remarks at the media briefing on COVID-19 - 11 March 2020. 2020. https://www.who.int/dg/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19—11-march-2020.
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COMMENTS

In Supporting Docs for this Modeling Scenario we have placed a Mathematica notebook 3-012-Mma-T-CoronoaVirusAerosols-TeacherVersion.nb and an attendant pdf version for those who do not read Mathematica files in which offer a complete analysis based on the information offered in the two references [2, 3] and come to the conclusion that with a reasonable ejection velocity for the virus aerosol that the distance the aerosol would travel would be 8.68 ft and in many cases further. Therefore, our analysis puts in question the advisory that it is sufficient to stand 6 feet apart to avoid the contagion with the virus as is advised by all public health agencies.