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STUDENT VERSION
Rocket Flight

Brian Winkel, Director SIMIODE
Cornwall NY USA

Abstract: We offer an opportunity to build a mathematical model using Newton's Second Law of Motion and a Free Body Diagram to analyze the forces acting on the rocket of changing mass in its upward flight under power and then without power followed by its fall to earth.

Keywords: rocket, flight, resistance, thrust, mass

Tags: first order, non-homogeneous, ordinary differential equation, separable, Newton's Second Law, Free Body Diagram, engineering, physics, directed, medium

STATEMENT

Suppose we have a small rocket with a full 100 liter fuel tank. Fuel (with a mass density of .98 kg/l) is burned at a steady rate of 3 liters per second and can provide a constant thrust force of 5900 Newtons as it burns. The rocket and fuel tank has a mass of 400 kg with no fuel in it. The shape design of the rocket causes a resistance proportional to its velocity during flight of 2 $v(t)$ where $v(t)$ is in m/s and the constant 2 for proportionality of resistance is 2 N/(m/s). We aim the rocket straight up and launch it . . . 5 . . . 4 . . . 3 . . . 2 . . . 1 . . . Launch . . . We have lift off!!!

1. How long will the rocket burn and hence provide thrust? We call this time interval the *burn period*.
2. Build a differential equation model for the powered flight (during the burn period) of the rocket based on all the External Forces and Newton's Second Law of Motion.

    Here are some issues to consider. Construct a Free Body Diagram - very important. See Figure 1 for hints. What are reasonable initial conditions? The mass $m(t)$ is now a function of time. What is that function? There is an external constant force due to thrust during the burn. Lots of stuff changing except the acceleration due to gravity, g.

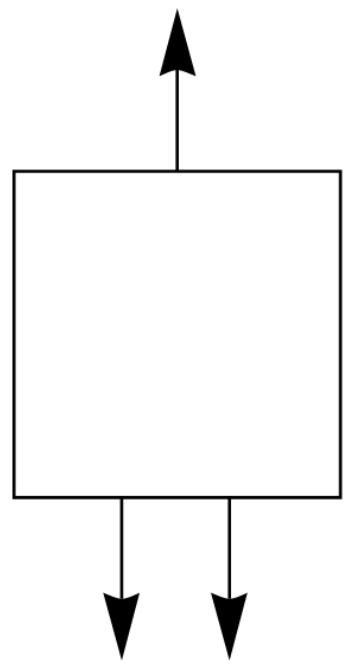

Figure 1. Outline of Free Body Diagram for upward rocket flight with fuel burn thrust looking for completion.

3. Solve the differential equation and plot the rocket's altitude over the burn period and until the rocket reaches ist apex or highest point. Also plot the velocity over the burn period.
4. Model what happens to the rocket after the burn period?
5. Determine maximum height of this rocket?
6. Plot the rocket's altitude over the free falling period. Also plot the velocity over the free falling period. We can assume the rocket turns around and heads downward with its front first as it did in the rising flight.
7. How long before it strikes the earth? That is, how long is the total flight? We may assume this is a vertical flight only.
8. With what velocity does the spent rocket strike the ground? Ouch!

In the above model we have moved away from constant coefficient differential equations. Moreover, we have introduced additional, important external forces (thrust and weight) to the model of the rocket. These forces will make our differential equation into a *non-homogeneous* differential equation.