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STUDENT VERSION
At what age do people get married?

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STATEMENT

There are many reasons someone may make the decision to or not to get married. Similarly there are many factors that cause people to get married at a wide variety of ages. Here's my story: I was fresh out of college when I got the phone call from my friend that I've had since preschool asking me to be in her wedding. I excitedly agreed. Fast forward a year: the wedding just happened and I'm telling my best friend from high school that she better wait a while until my bank account is replenished before she asks me to be in her wedding. It's only a month later when I get a phone call from her that begins "Please hear me out before you hang up," and I find myself getting ready to be in another wedding.

Is there a correlation? Is that when my friends were at the "common age" for (a first) marriage? Let's see if we can find an age at which people marry the most quickly; i.e. an age at which the fraction of people who are married increases the most rapidly.

Define the following functions:

- $p(t)$: the number of people in the United States who are t years old
- $M(t)$: the number of people in the United States who are t years old and have been married
- $m(t) := \dfrac{M(t)}{p(t)}$: the fraction of people of age t in the United States who have been married

We will be considering differential equations in terms of $m(t)$, because we are looking for the age at which $m(t)$ is increasing the fastest.

The Peer Pressure Model

We assume that a person's chances of getting married (in some small time interval Δt) is proportional to the fraction of people who are already married by that age; this can be considered peer pressure[1] .

1. We first assume that the rate of change of $m(t)$ is proportional to the number of interactions (the fraction of) people married (for a first time) by age t have with (the fraction of) people of age t who have not yet married.
   (a) Using our notation above, what fraction of people are married by age t? What fraction are unmarried at age t? How many interactions are there between these groups of people?
   (b) Write and solve a differential equation modeling the situation with these assumptions.
   (c) What is the long-term behavior of this solution? Does this make sense for the scenario? *Hint: Based on the situation, is the proportionality constant positive or negative? Why?*
   (d) This differential equation is autonomous.
      i. Find the equilibrium point(s).
      ii. Analyze the stability of the equilibrium point(s).
   (e) Your general solution to Problem 1b should have two unknown constants in it; the proportionality constant and another constant that arose when solving the first-order differential equation. Choose three different combinations of values for these constants. Plot the solution with these values plugged in.
      i. Do these graphs support your answers from Problem 1c and 1d?
      ii. What do you notice about solution at $t = 0$? Does this make sense?
   (f) One criticism of this model is that it assumes people experience that peer pressure to the same degree at every age. Come up with at least two other criticisms.

Let's try to compensate for the criticism mentioned in Problem 1f (that people probably don't experience peer pressure to the same extent at every age). We can do this by making a proportionality function $f(t)$ (rather than a proportionality constant), and leaving everything else unchanged.

2. Write and solve a differential equation (as much as possible) modeling the situation with these assumptions.

The first proportionality function used by G. Hernes [3] in an attempt to match numerical data was

$$f(t) = Ab^t$$

where $b < 1$ and A is the the initial fraction of people married.

3. How should the proportionality function $f(t)$ behave for the model to make sense? Therefore, why is this a reasonable guess?

4. Solve your differential equation from Problem 2 with this proportionality function.
5. Recall that A is the initial fraction of people married. Why doesn't this make sense, and what problem does it cause with your solution from Problem 4?

To work around this initial condition problem, A. J. Coale [2] made the following linear transformation of the age axis:

$$x := at - c.$$

The other axis was scaled to the number of people whose first marriage was at time x (i.e. our function $M(x)$). Upon doing this, it was found that the numerical data closely fits the curve

$$M(x) = e^{-e^{x}}.$$

6. Does this make sense given your solution in Problem 4? Explain and justify your reasoning.
7. Time to go back to the original question. Was this to be expected? Is there a "peak" age at which people get married?

REFERENCES

[1] Bender, E. 1978. *Introduction to Mathematical Modeling*. New York: John Wiley & Sons.

[2] Coale, A. 1971. Age Patterns of Marriage. *Population Studies*. 25(2): 193-214.

[3] Hernes, G. 1972. The process of entry into first marriage. *American Sociological Review*. 37: 173-182.