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STUDENT VERSION

Modeling Indoor Temperature

After Thanos Attacks

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STATEMENT

After Thanos made half the population vanish [1], weather patterns on Earth became much more stable to the point that, for modeling purposes, we can assume there is no change in the overall day-to-day weather pattern. We will assume in this model that 24 hours later, the weather will be identical to what it is now. Hence, we can model the outdoor temperature by a periodic function.

Pre-Activity

1. What is a periodic function? List at least three examples of periodic functions.

2. What is the period of a periodic function? Find the period for each function you listed in question 1.

3. We assume that the temperature 24 hours from now is identical to the temperature right now. What property does the temperature $T(t)$ have (in terms of questions 1 and 2)? How can you express this mathematically?

4. Does this make sense in the long-term (for example, 100 days later)? How could the model be changed to make more sense in the long-term?
Note: Lucky for us, this is reasonable for a few days and (as you'll soon see) that's all we really need.

Main Activity

As the outdoor temperature is periodic, we can write it as

$$T(t) = a + b\cos(\omega t) + c\sin(\omega t) \tag{1}$$

where t is time in hours. The first week we return to class in January, the lowest temperature of 35°F occurs at 4 A.M. and the highest temperature of 55°F occurs at 4 P.M. each day.

5. What is the smallest positive number ω that could be used in your model so that the temperature 24 hours later is identical to what it is now?
6. Use the high and low temperatures (and times) to solve for a, b, and c.

   Suggestion: Take $t = 0$ to be midnight, and assume that

   $$T(t) = a + d\cos(\omega\beta - \omega t). \tag{2}$$

   - What is the smallest positive value of β that makes the function have a maximum/minimum when it is supposed to?
   - What values for a and d make the function have the correct maximum/minimum value?
   - Use the identity

     $$\cos(\omega\beta - \omega t) = \cos(\omega\beta)\cos(\omega t) + \sin(\omega\beta)\sin(\omega t)$$

     to rewrite $T(t)$ in (2) into its original form (1) (i.e. from this identity, you can now find b and c).

Suppose that the furnace in your residence hall breaks at midnight one night. We are going to look at the temperature in your room, $y(t)$, over the next few days while the furnace is being repaired.

7. Since the furnace is off, the only influence on your room temperature is the "surrounding" outdoor temperature.

   (a) Assume that it was 75°F in your room when the furnace broke. Using Newton's Cooling Law and the equation you found for the outdoor temperature $T(t)$ (in the form of (1) above), write an initial-value problem to model the temperature in your room.
   Suggestion: Let $t = 0$ be midnight the night the furnace goes out.

   (b) Should the proportionality constant k in (the differential equation form of) Newton's Cooling Law be positive or negative? Explain your reasoning.

   (c) Assume that $|k| = 0.4$ (and use the sign of k from question 7b). Solve your initial-value problem from 7a with this proportionality constant; you may use technology.

(d) Graph the solution $y(t)$.

(e) You have medication in your room that is supposed to be kept above 45°F. Do you need to worry? Make some observations and discuss the ability of your solved solution to reflect reality.

(f) Describe the long-term behavior in words. At what time(s) is the temperature in your room the highest? Lowest? *Be precise.*

8. Use technology to solve the same initial-value problem, but assume that the temperature in the room when the furnace broke was

   (a) 80°F

   (b) 70°F

   (c) 65°F

   Plot all three solutions on the same graph. What do you notice?

9. Use technology to solve the same initial-value problem (assuming the room was initially 75°F), but assume that the proportionality constant is

   (a) $|k| = 0.1$

   (b) $|k| = 0.6$

   (c) $|k| = 0.9$

   (using the sign from question 7b).

   i Plot all three solutions on the same graph. What do you notice?

   ii Physically, the proportionality constant is related to how well the building is insulated. Does a lower or higher value of $|k|$ correspond to a well-insulated building? Justify your conclusion.

REFERENCES

[1] *Avengers: Infinity War*. Directed by Anthony Russo and Joe Russo, Marvel Studios, 2018.