Spring Design to Meet Specs at Minimum Costs

Spring Cost

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Class simulation followed by discussion.

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SIMIODE is funded by the National Science Foundation Division of Undergraduate Education.

Send questions to Director@simiode.org.

During this session we simulate a class activity on modeling with a differential equation. A full narrative on use along with materials for this session are available at

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Spring Design to Meet Specs at Minimum Cost

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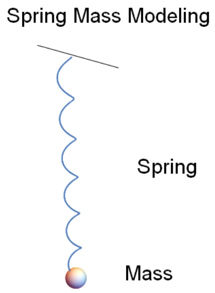
Spring Cost

We put together a model for determining spring constant in a commercial spring to meet certain specifications at low costs.

1. Model single spring used in some application.
2. Ascertain spring constant which gives overdamped solution.
3. Determine spring constant and minimum cost to meet specifications:
   * From same initial condition spring reaches given distance from static equilibrium in a given time.
   * For each set of specifications (a) given distance from static equilibrium and (b) given time render minimum cost.
4. Discussions

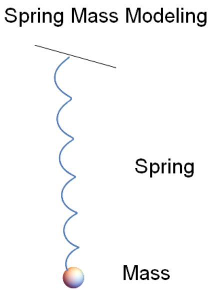
3-031-S-SpringCost **https://www.simiode.org/resources/7553**
3-031-T-SpringCost **https://www.simiode.org/resources/7554**

Now let us think of a vertically hanging spring with mass m at the bottom of the spring, whose displacement from its static equilibrium is measured by $y(t)$ in meters at time t in seconds.



Newton's Second Law of Motion says that the mass times the acceleration of that mass, $m \cdot y''(t)$, is equal to the sum of all the external forces acting on the mass.

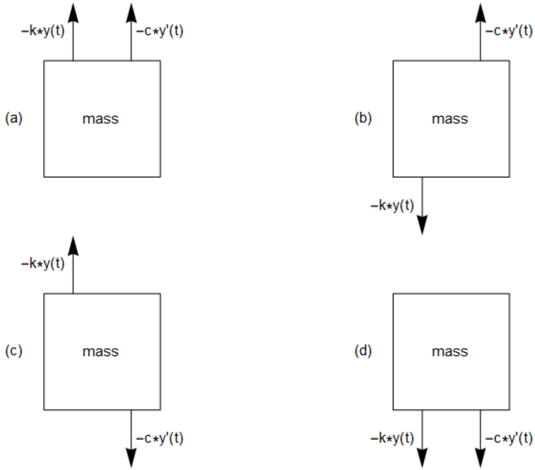
Consider spring mass system below.



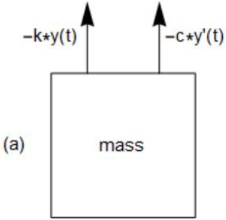
Place the mass at static equilibrium so the force due to gravity is balanced by force needed to pull the spring up from its stretched state. The spring is at rest.

If we now further extend the spring and release it what are the forces acting on mass? We display them in a Free Body Diagram.

Options for a Free Body Diagram. All constants, m, c, and k are positive and $y'(t) > 0$ and $y(t) > 0$ stand for downward velocity and position or expansion of the spring from static equilibrium. $y'(t) < 0$ and $y(t) < 0$ stand for upward velocity and position or compression. Choose correct Free Body Diagram!



From correct Free Body Diagram build a differential equation of the motion of the mass using Newton's Second Law of Motion.



$$m \cdot y''(t) = \rule{5cm}{0.4pt}?$$

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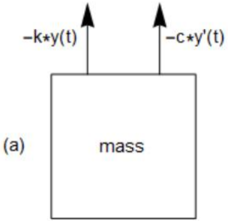


$$m \cdot y''(t) = \underline{\hspace{6cm}}?$$

$$m \cdot y''(t) = -cy'(t) - ky(t) \tag{1}$$

Now using Mathematica we see the general solution of (1) with initial conditions, say $y(0) = 0.4.$ and $y'(0) = 0$

$$
\begin{aligned}
y(t) \quad = \quad & \frac{0.2c\exp\left(\frac{t\sqrt{c^2-4km}}{m} - \frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}\right)}{\sqrt{c^2-4.km}} \\
& +0.2\exp\left(\frac{t\sqrt{c^2-4km}}{m} - \frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}\right) \\
& -\frac{0.2ce^{-\frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}}}{\sqrt{c^2-4.km}} + 0.2e^{-\frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}}.
\end{aligned}
$$

What common expression do we see in all terms?

What common expression do we see in all terms?

$$\sqrt{c^2 - 4km}$$

If $c^2 - 4km > 0$ all exponential forms in our solution, e.g.,

$$e^{\left(\frac{t\sqrt{c^2-4km}}{m} - \frac{t\left(\sqrt{c^2-4km}+c\right)}{2m}\right)}$$

exhibit exponentially decay and solution goes to zero.

This means the spring displacement will return to 0, BUT with no oscillation for there would be no sine or cosine terms as in the case when $c^2 - 4km < 0$.

We consider a spring with $m = 1.2$ kg, $c = 46.4$ N/(m/s).

k N/m is to be determined in order to meet certain specifications. Thus we have a model of our specific spring mass system as a differential equation with initial conditions (2):

$$1.2y''(t) + 46.4y'(t) + ky(t) = 0\,,\ \ y(0) = 0.4\,,\ y'(0) = 0\,. \qquad (2)$$

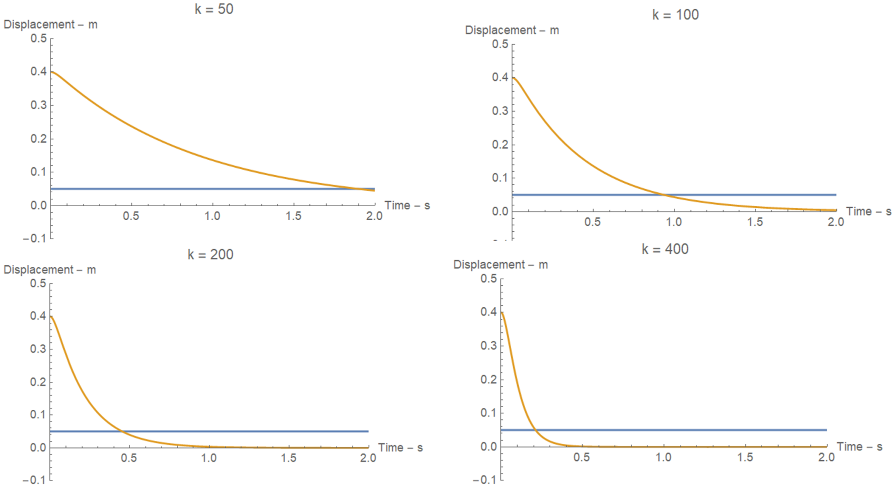
We have been tasked to determine spring values k so that the spring meets certain specifications.

Further, we have been advised that to produce a spring with spring constant k there is a cost of $\text{Cost}(k) = 13.70 + 0.01k^2$.

So what are these specification?

We examine some cases for values of $k = 50, 100, 200, 400$.

All cases where spring is overdamped, i.e. $c^2 - 4km > 0$.



Specifications

We seek a value of k such that for the spring design in (2)

$$1.2y''(t) + 46.4y'(t) + ky(t) = 0\,, y(0) = 0.4\,, y'(0) = 0\,, \qquad (2)$$

1. spring is overdamped,
2. spring returns to within a meters of static equilibrium of 0,
3. spring does so within b seconds from time $t = 0$.

Further, we wish to know the cost of such a spring to meet these specifications for all combinations of a and b which are reasonable.

For $a = 0.2$ m and $b = 0.5$ s draw a sketch of displacement of the spring over time and explain in the sketch the role of a and b.

For this sketch there is no need to ascertain k, just a sketch.

Assignment

1. Explain what it means for a spring to return to within a m of its static equilibrium of 0 in b seconds from $t = 0$.
2. Using cost function, $\text{Cost}(k) = 13.70 + 0.01k^2$ find the cheapest spring with spring constant k which returns to within $a = 0.1$ m of its static equilibrium of 0 in $b = 0.5$ seconds from time $t = 0$.
3. Provide others a tool through which they can determine, generally, the cost function for a spring which is overdamped, returns to within a m of its static equilibrium of 0, and does so within b seconds from time $t = 0$.
4. Provide a representation of level curve pricing in which for a fixed cost, say \$600, all possible spring configurations (i.e. returns to within a m of its static equilibrium of 0 and does so within b seconds from time $t = 0$) are displayed.

CONGRATULATIONS!!!

We went from

- rebuilding spring-mass model with differential equation,
- to solving equation with spring constant k as parameter,
- to determining parameter values for overdamped solution,
- to using our model to determine spring constant which meets specifications on distance from static equilibrium in set time,
- to determining cost of resulting spring,
- to custom designing spring for given specifications,
- to examining all spring configurations for fixed costs.

Discussion, Questions, Ideas please . . .

Contact Director@simiode.org

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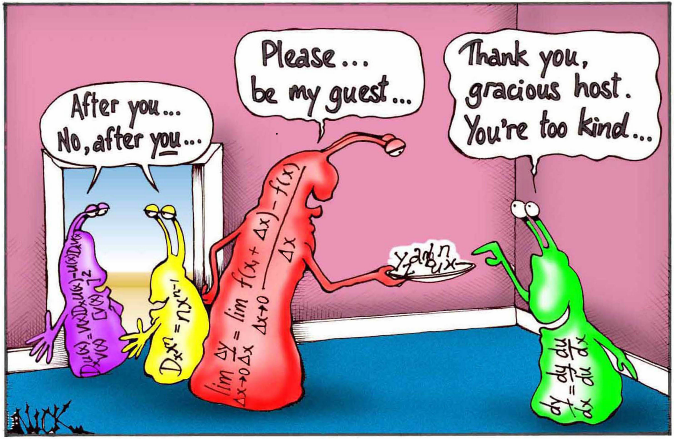
OR just search “SCUDEM” in Google.

Fall 2020 with Challenge Saturday due date 14 November 2020.

Features of SCUDEM V 2020 include

- teams of three high school, home school, individuals, or undergraduate students - undergraduate level or lower
- teams from same institution or **assembled by SIMIODE from students around the world** on each team
- mentor/coach faculty engage with teams and fellow mentors/coaches
- mentoring period 1 September 2020 through 23 October 2020
- three problems from physics/engineering, chemistry/life sciences, and social sciences/humanities released on 23 October 2020
- students prepare 10 minute video presentation and upload to YouTube by Challenge Saturday, 14 November 2020
- faculty from around the world judge and give feedback
- Outstanding, Meritorious, and Successful awards
- SIMIODE posts ALL student team submissions and essay by problem poser on student submissions.

Discussions and Questions



Deferential equations.

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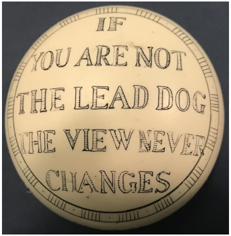

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