26 January 2021

SIMIODE is proud to announce a Preview Version of its forthcoming online text, *Differential Equations: A Toolbox For Modeling The World*.

Developed and written by the distinguished teacher and author, Dr. Kurt Bryan, Rose-Hulman Institute of Technology, Terre Haute IN USA, this text takes a modeling first and throughout approach to motivate the study and learning of differential equations in the spirit of SIMIODE, while linking to many SIMIODE Modeling Scenarios and other activities.

The text will be available in its Preview Version in January 2021 and will be freely offered immediately to those who register for **SIMIODE EXPO 2021**, 12-13 February 2021, which includes a chance to meet the author in a Panel Session during the conference.

Faculty who view this Preview Version will have an opportunity to adopt this online text for their students for Fall 2021 use in their teaching for a very modest cost per student, of $45 per copy. We seek comments and feedback on the text at any time. Send them along to **Director@simiode.org**.

Revisions, additional worked exercises, appendices, and some new material will be added with the complete text available for adoption c. 15 May 2021.

*Differential Equations: A Toolbox For Modeling The World* puts applications and modeling front and center in an introduction to ordinary differential equations. In taking this approach we do not skimp on or skip over the mathematics but use applications to motivate almost every subject and technique. The mathematics presented is interwoven with modeling to drive both the mathematics and understanding of the application under study and to make the case that differential equations provide a powerful, indispensable toolbox for describing the world.

We present some unconventional, but important topics not usually offered in introductory texts: dimensional analysis, parameter estimation, a brief introduction to control theory via Laplace transforms, stiff systems of differential equations, and a more thorough treatment of electrical circuits. The text includes numerous exercises, including inline ``Reading Exercises,” as well as a section of more extensive modeling projects at the end of each chapter, many based on published SIMIODE projects, and many new activities. Several projects include data sets for experimentation and model validation.

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