Adapting SIMIODE Projects to an Online ODE Course

Michael A. Karls
Abstract

• In Fall 2017 I incorporated several SIMIODE projects into a traditional ODE course.

• For Fall 2020, I used a similar approach, however this time the course was taught in an asynchronous online format.

• Just as was done the first time, I wrote accompanying project assignments to help guide students through the SIMIODE projects.

• I will look at examples of these accompanying projects, compare how they worked or didn’t work in each class setting, and consider what modifications could be made for future versions of the course.
What Has Been Done ...

• Since 1993, I have been teaching an introductory ODE course at Ball State.

• The course, MATH 374 Differential Equations can be considered “classic” or “traditional” based on the material covered as well as the method of delivery.
Textbook(s) Used

• The first few times I taught the course, the textbook used was *Elementary Differential Equations*, by Rainville, Bedient, and Bedient.

• I found that this book worked well, mainly because each section covers exactly one topic and the exposition is concise, easy to follow, and complete.
Textbook(s) Used

- About 18 years ago, we switched to Boyce and DiPrima’s *Elementary Differential Equations and Boundary Value Problems* text.
  - Textbooks for most lower-level courses are chosen by committee.
  - I’ve adapted textbook readings to my original notes.
  - Boyce and Diprima provides a nice amount of extra detail to supplement class lectures.
  - The only downside is that one has to “jump around” in the text to match class notes.
Textbook(s) Used

• For Fall 2020 I switched to an online textbook found at SIMIODE’s Free Online Texts resources web page.

• I used Jiří Lebl's Notes on Diffy Q's: Differential Equations for Engineers
  • Lebl designed the text to be a “drop-in replacement” for courses that use Edwards and Penney, Differential Equations and Boundary Value Problems: Computing and Modeling, or Boyce and DiPrima's Elementary Differential Equations and Boundary Value Problems.
  • This was supplemented with William Trench's free book Elementary Differential Equations with Boundary Value Problems, also available at SIMIODE’s Free Online Texts resources web page.
SIMIODE Background Material

- SIMIODE Starter Kit (https://www.simiode.org/starterkit)
- Sample SIMIODE Course Syllabus
- Testimonials and Reflections by Colleagues
  - DIFFERENTIAL EQUATIONS AT MANHATTAN COLLEGE PERSONAL ACCOUNT by Rosemary Farley
Choosing and Assigning Projects (2017)

• My plan was to choose projects that would align with the material covered in class.

• Assign one project each Friday, due the following week, with Friday’s class time used as a lab day (in lieu of working on homework examples).

• Projects would be due the following Friday at midnight, submitted via Blackboard.
Choosing and Assigning Projects (2017)

• No projects would be assigned in an Exam week or during the weeks with Final Presentations at the end of the semester.

• This meant approximately eight project assignments!

• Since I’d never used this approach, my plan was to see where we were each week and choose a project accordingly.
# Course Outline (Fall 2017)

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Examples of Differential Equations, Definitions, Families of Curves</td>
<td>Ant Tunnel Building (due week 2)</td>
</tr>
<tr>
<td>2</td>
<td>Families of Solutions, Geometric Interpretation; Isoclines and Direction Fields, Existence and Uniqueness</td>
<td>M&amp;M - Death and Immigration (due week 4)</td>
</tr>
<tr>
<td>3</td>
<td>Homogeneous Functions; Equations with Homogeneous Coefficients, Separation of Variables</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Differentials, Exact Equations; Linear Equations</td>
<td>Potato Cooling, Oil Slick (due week 6)</td>
</tr>
<tr>
<td>5</td>
<td>Elementary Applications; Autonomous Equations, Phase Lines</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td><strong>Exam 1</strong>, Linear Differential Equations</td>
<td>M&amp;M - Disease Spread (due week 8)</td>
</tr>
</tbody>
</table>
# Course Outline (Fall 2017)

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Matrix Theory Review, The Wronskian and Linear Independence</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>General Solution – Homogeneous and Non-Homogeneous; Differential Operators, Laws of Operation</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Properties of Differential Operators; The Auxiliary Equation: Distinct Roots</td>
<td>Data to Differential Equations (due week 11)</td>
</tr>
<tr>
<td>10</td>
<td>Auxiliary Equation with Repeated Roots, Complex Valued Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Auxiliary Equation with Complex Roots, Hyperbolic Functions</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Construction of a Homogeneous Equation from a Specified Solution, The Method of Undetermined Coefficients; Reduction of Order</td>
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</tr>
<tr>
<td>12</td>
<td>Variation of Parameters, Exam 2</td>
<td></td>
</tr>
</tbody>
</table>
# Course Outline (Fall 2017)

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>First Order Systems, Solution of a First Order System; Matrix Representation of a System, First Order Systems Revisited</td>
<td>Bubbles of Beer (due week 15)</td>
</tr>
<tr>
<td>14</td>
<td>Complex Eigenvalues; Repeated Eigenvalues</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Non-homogeneous Systems</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Exam 3, Laplace Transforms, Final Presentations</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Final Presentations, Final Exam</td>
<td></td>
</tr>
</tbody>
</table>
## Project Details

<table>
<thead>
<tr>
<th>Project</th>
<th>Keywords</th>
<th>Tags</th>
<th>Brief Topic Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ant Tunnel Building</td>
<td>ant, tunnel, derivative, integrate, doubling</td>
<td>first order, differential equation, model</td>
<td>Simple approach to create differential equation model. First order equation solvable via direct integration.</td>
</tr>
<tr>
<td>M&amp;M - Death and Immigration</td>
<td>difference equation, differential equation, parameter estimation, simulation, data collection</td>
<td>first order, equilibrium value, M&amp;M's, simulation, data, death, immigration, modeling</td>
<td>Simulation with M&amp;M's for population model with immigration and parameter estimation. Difference and differential equations, exponential growth.</td>
</tr>
<tr>
<td>Potato Cooling, Oil Slick</td>
<td>cooling, potato, first order, differential equations, linear; oil slick, data, incomplete, problem solving, spread</td>
<td>model, fitting, line, assumptions, difference equation, differential equation, linear, first order, non-homogeneous</td>
<td>Modeling a cooling potato; Modeling growth of oil slick - challenging problem with poor data. Separable or linear equations.</td>
</tr>
</tbody>
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<tr>
<td>M&amp;M - Disease Spread</td>
<td>disease, spread, data collection, logistic, epidemic</td>
<td>sum of square errors, parameter estimation, Newton’s Law of Cooling; first order, differential equation, parameter estimation, m&amp;m, simulation, data analysis</td>
<td>Using a grid and M&amp;M candies, to produce data from a simulation of the spread of disease. Estimate parameters in a differential equation model, logistic growth.</td>
</tr>
<tr>
<td>Data to Differential Equations</td>
<td>spring, mass, data fitting, parameter estimation, total distance</td>
<td>second order, linear, differential equations, parameter estimation, numerical differentiation</td>
<td>Second order linear equation. Estimate the damping coefficient and spring constant from collected spring-mass system data. The data is presented as total distance traveled. Numerical estimates.</td>
</tr>
<tr>
<td>Modeling Bubbles of Beer</td>
<td>First-order, nonlinear, system, numerical methods, beer, bubble, Newton’s Second Law, Ideal Gas Law, drag force.</td>
<td>Hadamard, Stokes, sphere, surface area, volume, mole, parameter, Mathematica, Manipulate, MatLab, M-file.</td>
<td>Set up and numerically solve a nonlinear ODE system. Verify the model via data collected from a bubble rising in a glass of beer.</td>
</tr>
</tbody>
</table>
Project Assignments (2017)

- After the students worked on the first SIMIODE project (Ant Tunneling) in class, I decided to write up an accompanying project assignment.

- This was based on our discussions in class.

- This assignment was designed to help guide students through the SIMIODE project.
Project Assignments (2017)

• After the students worked on the first SIMIODE project (Ant Tunneling) in class, I decided to write up an accompanying project assignment.

• This was based on our discussions in class.

• This assignment was designed to help guide students through the SIMIODE project.

*Done in-person in a classroom.*
Project 1 (Fall 2017)

1. Read the Ant Tunnel Building Handout.

2. Answer parts (a) – (h) from the handout. For part (a), use \( T(x) = kx \) and \( T(x) = kx + \alpha \), plus any other functions you created.

3. For the differential equation model obtained from the difference equation \( T(x+h) - T(x) = \alpha x h \), does the solution to this model seem reasonable? Why or why not? How could we verify this model is valid?

4. Show how one can arrive at the difference equation \( T(x+h) - T(x) = \alpha x h \). Hint: Assume for fixed \( x \), \( T(x+h) - T(x) \) is proportional to \( h \). Also assume for fixed \( h \), \( T(x+h) - T(x) \) is proportional to \( x \).
Project 2 (Fall 2017)

• Read the M&M – Death and Immigration Handout.

• Following the instructions in the handout on pages 1 and 3, perform the experiments for M&M Death and M&M Death and Immigration and record your results in Tables 1 and 2.
  • Information from these tables will be put into your work turned in for this project!

• Choose an integer from 6 – 16, record this number, and repeat the M&M Death and Immigration experiment with your chosen number instead of 10 for the number of M&M’s that immigrate into the population at each iteration.
  • Record your results in a third table, Table 3, set up the same as Tables 1 and 2.
  • Make a copy of this third table with the recorded data to share with a classmate – put your name on the table, but DO NOT let your classmate know your choice of immigration number.
  • The information from Table 3 that you receive from a classmate will be put into your work turned in for this project!
Project 2 (Fall 2017)

- Read the M&M – Death and Immigration Handout.

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*Done in-person in a classroom.
Project 2 (Fall 2017)

- In class, I asked you to try to find functions \( a(n) \) and \( b(n) \) to model the data you collected for each of the first two experiments.

- If you were not able to do so, that’s ok – this is hard to do directly.

- As we discussed in class (and in the handout), another way to approach this is to find a relationship between the number of M&M’s at a given iteration in terms of the number of M&M’s at the previous iteration.

- We came up with the recurrence relation \( b(n+1) = 0.5 \, b(n) + 50 \) for \( n \geq 1 \), as a suggested model for the second experiment.

- Note that this is equation (1) on page 5 of the handout.

- We also discussed the need for a starting point or initial condition for our model, which we agreed should be \( b(0) = 50 \).
Project 2 (Fall 2017)

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Project 2 (Fall 2017)

1. Find a recurrence relation of the form \( a(n+1) = \)___________ for \( n \geq 1 \), to model the first experiment, with appropriate initial condition for \( n = 0 \).

2. Create a table to display the information from Table 1 as well as a third column with the outputs for your model for each choice of integer \( n \). Repeat for the information from Table 2. 

Include these tables you create in your answer for this question. Compare your models for Experiments 1 and 2 to the data you collected. How well do they agree? Justify your answer mathematically.

3. Solve the recurrence relation for the second experiment, using the suggested Mathematica command from the handout. This can also be done with Wolfram Alpha. Does your answer agree with that in the handout? Why or why not?

4. Solve the recurrence relation you found for the first experiment, using Mathematica or Wolfram Alpha.
Project 2 (Fall 2017)

1. Find a recurrence relation of the form $a(n+1) = \underline{\quad}$ for $n \geq 1$, to model the first experiment, with appropriate initial condition for $n = 0$.

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4. Solve the recurrence relation you found for the first experiment, using Mathematica or Wolfram Alpha.
Project 2 (Fall 2017)

5. Repeat questions 3 and 4 by hand (i.e. without the use of the \texttt{RSolve} command).

6. On page 7 of the handout, there is a derivation of a differential equation associated with the second experiment, which leads to another model for the second experiment, namely the \textit{initial value problem} (IVP), (9). Solve the IVP (9) using the suggested Mathematica command in the handout. This can also be done with Wolfram Alpha. Does your answer agree with the solution (10) in the handout? Why or why not?

7. Solve IVP (9) by hand (i.e. without the use of the \texttt{DSolve} command).

8. Using an argument similar to that used to obtain the differential equation (and IVP) for the second experiment, find an IVP to model the \textit{first experiment}. Solve this second IVP.
9. Set up a model for the data you get from a classmate for the third experiment, involving the function $c(n)$. Note that there will be an unknown parameter in this model – use the symbol $k$ for this parameter. Be sure to include the name of the classmate who gave you the data in the answer to this question!

10. Watch the following YouTube video on parameter estimation via Excel: https://www.youtube.com/watch?v=mEthE6Hia-k&feature=youtu.be. Use the Excel Solver to determine the parameter $k$ for your model in question 9 to best fit the model for the data provided by your classmate. Include a copy of your work done in Excel – save this file as LastnameFirstname374Project2.xlsx.

11. Create a table to display the information from Table 3 as well as a third column with the outputs for your model found in question 10 for each choice of integer $n$. How well does your model fit the data? Justify your answer mathematically. Based on your model, guess the value of $k$ chosen by your classmate!
9. Set up a model for the data you get from a classmate for the third experiment, involving the function $c(n)$. Note that there will be an unknown parameter in this model – use the symbol $k$ for this parameter. Be sure to include the name of the classmate who gave you the data in the answer to this question!

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11. Create a table to display the information from Table 3 as well as a third column with the outputs for your model found in question 10 for each choice of integer $n$. How well does your model fit the data? Justify your answer mathematically. Based on your model, guess the value of $k$ chosen by your classmate!

*Done in-person in a classroom.*
Challenges for Fall 2020

• Due to the Covid-19 pandemic, many courses that would typically be face-to-face were offered in an online format.

• Almost all online courses (including MATH 374) were asynchronous (i.e. no set meeting time).

• All meetings (including office hours) were to be conducted remotely via WebEx or Zoom.

• Proctoring for Exams via Respondus Lockdown Browser was available (I did not use this for my course).

• Many instructors received training for online instruction in Summer 2020.
Revised Course for Fall 2020

• I had already run the course in Spring 2018 as an independent study, so my plan was to use a version of this format.

• This was also based on my experience of our university shifting to all online mid-term in Spring 2020.

• Plan:
  • Post approximately two PowerPoint lectures per week with readings and homework from Lebl’s text, supplemented by other material as needed.
  • Post the same projects as in Fall 2017, modified as needed.
  • Projects would be posted at 8 am on a Wednesday, due at 11:59 pm on a Friday, usually two weeks later.
  • Projects would count for 37.5% of course grade.
  • Exams would count for 25% of course grade.
  • These weights are swapped from Fall 2017.
  • *This meant that take-home work counted for 75% of the course grade.*
Working with Students on Projects (Fall 2020)

• Virtual office hours were scheduled for MW 2-2:50 pm via optional WebEx meetings, or by appointment.

• Projects were discussed mainly via sharing screens to look at students’ work, discuss ideas, provide insight, compare results, etc.

• I also used MS-Word to share mathematical ideas during these meetings ... some of my colleagues have used portable document cameras, iPads with Apple Pencils, webcams, etc.

• Another means to discuss the course was via email.
Canvas Discussion Boards

- Some questions and answers for Projects were posted on Discussion Boards in Canvas.
- For each project, I started the project’s Discussion Board with a question or hint, adding more as needed.
- This was initiated after posting the first project.
Project 1 (Fall 2020)

1. Read the Ant Tunnel Building Handout.

2. Answer parts (a) – (h) from the handout. For part (a), use $T(x) = kx$ and $T(x) = kx + \alpha$, plus any other functions you created.

3. For the differential equation model obtained from the difference equation $T(x+h) - T(x) = \alpha x h$, does the solution to this model seem reasonable? Why or why not? How could we verify this model is valid?

4. Show how one can arrive at the difference equation $T(x+h) - T(x) = \alpha x h$. Hint: Assume for fixed $x$, $T(x+h) - T(x)$ is proportional to $h$. Also assume for fixed $h$, $T(x+h) - T(x)$ is proportional to $x$. 
Project 1 (Fall 2020)

- For this first project, I tried to use the “same” approach as was used for Fall 2017.

- Right away, I realized that there would be a “problem” with this approach.

- Students wouldn’t get the opportunity to work on the SIMIODE Ant Tunneling project or discuss various aspects of the problem together in a classroom setting.

- My “solution” to this was to post the following to a Project 1 Discussion Board in Canvas.
Project 1 Discussion (initial)

• For part (a), these are functions I thought of for $T(x)$ ... the time for an ant to build a tunnel of length $x$.
  • $T(x) = kx$
  • $T(x) = kx + \alpha$

• Are these appropriate candidates for $T(x)$? Why or why not?

• Based on working with students in WebEx office hour meetings on the project, I followed up with more discussion in Canvas!
Project 1 Discussion (follow-up)

• Here's a hint for question 4:

• For fixed x, T(x+h) - T(x) is proportional to h means that if we fix x, T(x+h) - T(x) = kh, for some constant k. "Fixed x", means that for example, T(1+h) - T(1) = k1h for some constant k1. If x = 2, we'd have T(2+h) - T(2) = k2h for some possibly different constant k2. So, in general, k depends on x.

• Similarly, for fixed h, T(x+h) - T(x) is proportional to x means that if we fix h, T(x+h) - T(x) = mh, for some constant m. "Fixed h", means that for example, T(x+1) - T(x) = m1x for some constant m1. If h = 2, we'd have T(x+2) - T(x) = m2x for some possibly different constant m2. So, in general, m depends on h.
Project 2 (Fall 2020)

- This project relies on collecting data from M&M’s provided to students in a classroom setting.

- Besides removing the “fun” from doing this experiment in person, there were clear logistical challenges, as I couldn’t provide M&M’s nor expect students to have (or purchase) their own supplies.

- Fortunately, virtual experiments are provided in the SIMIODE materials!

- The next few slides show how the project was adjusted for online instruction.
*Revisions for asynchronous online setting.

**Project 2 (Fall 2020)**

- Read the M&M –Death and Immigration Handout
  - [1-1-S-MM-DeathImmigration-StudentVersion.pdf](#)
  - *Do NOT answer any of the questions in this handout. For this project, please answer questions 1-11, given below.*

- Following the instructions in the handout on pages 1 and 3, perform the experiments for **M&M Death** and **M&M Death and Immigration** and record your results in Tables 1 and 2.
  - Information from these tables will be put into your work turned in for this project!
  - *If you do not have M&M’s available, virtual experiments with M&M’s are provided in the included PDF files (note that the initial number of M&M’s is 55 for these virtual experiments):*
    - [1-1-MandM-PureDeathSimulationImageData.pdf](#)
    - [1-1-MandM-Death-ImmigrationSimulationImageData.pdf](#)
Project 2 (Fall 2020)

- I chose an integer $k$ from 6–16 and repeated the M&M Death and Immigration experiment with my chosen number $k$ instead of 10 for the number of M&M’s that immigrate into the population at each iteration.

- Here is a table of my results, Table 3, set up the same as Tables 1 and 2.

- The information from Table 3 will be put into your work turned in for this project!

<table>
<thead>
<tr>
<th>Iteration</th>
<th># M&amp;M’s at start of iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
</tr>
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<td>3</td>
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<td>8</td>
<td>15</td>
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<tr>
<td>9</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3: Modeling death and immigration of M&M’s (# immigrants, $k$, is unknown)

*Revisions for asynchronous online setting.*
Project 2 (Fall 2020)

- Can you think of functions $a(n)$ and $b(n)$ to model the data you collected for each of the first two experiments?

- If you are not able to do so, that’s ok – this is hard to do directly.

- As discussed in the handout, another way to approach this is to find a relationship between the number of M&M’s at a given iteration in terms of the number of M&M’s at the previous iteration.

- A suggested model for the second experiment is the recurrence relation $b(n+1) = 0.5 \, b(n) + 10$ for $n \geq 1$.

- Note that this is equation (1) on page 5 of the handout.

- We also need a starting point or initial condition for our model, which in this case will be $b(0) = 50$, since we started with 50 M&M’s for this experiment.

- (For the virtual experiment, $b(0) = 55$.)
Project 2 (Fall 2020)

1. Find a recurrence relation of the form $a(n+1) = \_\_\_\_\_\_\_\_\_\_$ for $n \geq 1$, to model the first experiment, with appropriate initial condition for $n = 0$.

2. Create a table to display the information from Table 1 as well as a third column with the outputs for your model for each choice of integer $n$. Repeat for the information from Table 2. Include these tables you create in your answer for this question. Compare your models for Experiments 1 and 2 to the data you collected. How well do they agree? Justify your answer mathematically.

3. Solve the recurrence relation for the second experiment, using the suggested Mathematica command from the handout. This can also be done with Wolfram Alpha. Does your answer agree with that in the handout? Why or why not?

4. Solve the recurrence relation you found for the first experiment, using Mathematica or Wolfram Alpha.
Project 2 (Fall 2020)

5. Repeat questions 3 and 4 by hand (i.e. without the use of the `RSolve` command).

6. On page 7 of the handout, there is a derivation of a differential equation associated with the second experiment, which leads to another model for the second experiment, namely the initial value problem (IVP), (9). Solve the IVP (9) using the suggested Mathematica command in the handout. This can also be done with Wolfram Alpha. Does your answer agree with the solution (10) in the handout? Why or why not?

7. Solve IVP (9) by hand (i.e. without the use of the `DSolve` command).

8. Using an argument similar to that used to obtain the differential equation (and IVP) for the second experiment, find an IVP to model the first experiment. Solve this second IVP.
9. Set up a model for the data you got from me for the third experiment, involving the function $c(n)$. Note that there will be an unknown parameter in this model—use the symbol $k$ for this parameter.

10. Watch the following YouTube video on parameter estimation via Excel: https://www.youtube.com/watch?v=mEthE6Hia-k&feature=youtu.be. Use the Excel Solver to determine the parameter $k$ for your model in question 9 to best fit the model for the data provided by your classmate. Include a copy of your work done in Excel—save this file as LastnameFirstname374Project2.xlsx.

11. Create a table to display the information from Table 3 as well as a third column with the outputs for your model found in question 10 for each choice of integer $n$. How well does your model fit the data? Justify your answer mathematically. Based on your model, guess the value of $k$ that I chose!
Project 2 Discussion (initial)

• Each M&M has an "M" on one side. If you gently toss 50 (or 55) M&M's onto a paper plate, how many would you expect to have the "M" facing up?

• The next item was posted after working with students in virtual office hours.
Project 2 Discussion (first follow-up)

• For the virtual experiments provided, based on what is described in the handout for the experimental procedures, consider Generation 1 to be Iteration 0 and use this for Tables 1 and 2:

• Table 1:
  - n     # M&M's at start of iteration
  - 0     55
  - 1     27

• Table 2:
  - n     # M&M's at start of iteration
  - 0     55
  - 1     40

• This will match with how I collected data with my own M&M’s and with the models in Figure 1 and Table 3 in the handout.
Project 2 Discussion (second follow-up)

- There is a recurrence relation for each experiment.

- For Experiment 2 (death and immigration), the recurrence relation is: \( b(n+1) = 0.5b(n) + 10 \) for \( n \geq 1 \).

- For Experiment 1 (pure death), the recurrence relation is \( a(n+1) = 0.5a(n) \) for \( n \geq 1 \).

- Note: These also work for \( n = 0 \).
## Project “Adjustments” – Classroom vs. Remote

<table>
<thead>
<tr>
<th>Project</th>
<th>Fall 2017</th>
<th>Fall 2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ant Tunnel Building</td>
<td>Held Class Discussion</td>
<td>Posted Canvas Discussion</td>
</tr>
<tr>
<td>M&amp;M - Death and Immigration</td>
<td>M&amp;M experiments in class, classmates shared collected data with classmates</td>
<td>Virtual M&amp;M’s experiments provided, shared my data with class</td>
</tr>
<tr>
<td>Potato Cooling, Oil Slick</td>
<td>Newton’s Law of Cooling introduced in class</td>
<td>Newton’s Law of Cooling referred to in text</td>
</tr>
<tr>
<td>M&amp;M - Disease Spread</td>
<td>M&amp;M’s and grid handed out in class</td>
<td>Virtual M&amp;M’s experiment provided</td>
</tr>
<tr>
<td>Data to Differential Equations</td>
<td>Discussed approach using Mathematica in class, posted sample commands in two files that were revised based on student input</td>
<td>Posted sample Mathematica file, provided leading questions and detailed hints based on WebEx meetings with students</td>
</tr>
<tr>
<td>Modeling Bubbles of Beer</td>
<td>Provided hints in class as the project progressed, based these on student feedback in class</td>
<td>Provided pointed hints, followed by detailed hints based on student’s submitted work</td>
</tr>
</tbody>
</table>
My Observations (Fall 2017)...

- One of the main obstacles for students (besides the mathematical concepts) was the software needed.
- Students were more familiar with Excel (most had not encountered the Solver).
- Mathematica was a challenge for many students.
- This is typical for students in most courses.
My Observations (Fall 2017)...

- Initially, students struggled with the projects.
- As the semester progressed, they “got the hang” of the projects.
- I had to provide liberal hints, especially with the projects involving Mathematica.
- Overall, students seemed to do better in the course than in previous years.
My Observations (Fall 2020)...

• One of the main obstacles for students (besides the mathematical concepts) was the software needed.

• Students were more familiar with Excel (most had not encountered the Solver).

• Mathematica was a challenge for many students.

• This is typical for students in most courses.

• Wolfram Alpha caused some unexpected problems
  • Several students used this instead of Mathematica
  • Complex Mathematica commands entered in Wolfram Alpha didn’t work
  • It seems that these did work in previous years
My Observations (Fall 2020)...

• Students seemed to struggle with the projects not only initially, but throughout the semester.

• This is hard to gauge completely, as there was little or no interaction with approximately 75% of the class.

• I had to provide liberal hints, especially with the projects involving Mathematica.

• One neat result was that for Project 6, some of the students came up with a method to minimize error in Mathematica for numerical solutions to an ODE system that I (and colleagues) have never seen!
What to try next time …

• Continue with the SIMIODE projects.

• Spend more time on projects in class.

• Perhaps try video lectures posted ahead of time.

• If online, offer in a synchronous format.

• If at all possible, offer the course in a classroom setting.

• Ball State has Interactive Learning Spaces that may be more conducive to this setting than a traditional classroom.

http://cms.bsu.edu/about/administrativeoffices/educationalexcellence/services/learningspacesinitiative