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STUDENT VERSION
Bad Vibrations: Modeling a Building During an Earthquake
Part II: With Damping

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SCENARIO DESCRIPTION

In Part I [6], we analyzed how ground movement can cause vibrations in a building. Sometimes these resulting vibrations can end up being too big, resulting in discomfort of the people occupying the building or, in the worst case scenario, the building collapsing. To avoid this, damping effects are designed into a structure to attempt to minimize vibrations. Your goal here is to determine how much friction/damping should be designed into a building to keep the roof from moving too far (which would result in the entire building collapsing) when it undergoes minor vibrations from a small earthquake.

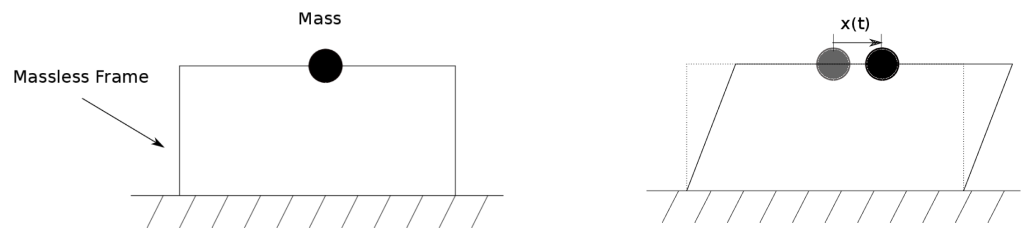
ACTIVITY 1 – NO VIBRATIONS

The process by which free vibrations diminish is called **damping**. Energy of the vibrating system is released by various mechanisms (usually several at once). In a vibrating building, friction at connections and the formation of microcracks are just two possible ways energy is dissipated. It is extremely difficult (and at times impossible) to describe each mechanism individually.

PART A – Viscous Damping

Here we idealize the damping forces into a single term:

$$f_D = \gamma x'$$



(a) Considering the roof as a point mass at the center (b) Displacement of the roof is measured by $x(t)$

Figure 1: Motion of the center of mass of the roof of a one-story building

where γ is chosen so that the change of the vibration energy is equal to the change of energy caused by all the damping mechanisms. This is referred to as **equivalent viscous damping** and γ is called the **viscous damping coefficient**. Considering this damping force also acting on our building, our model now is

$$mx'' + \gamma x' + kx = 0, \qquad x(0) = x_0, \;\; x'(0) = x_1. \tag{1}$$

Problem 1. What units should the constant γ have for the differential equation (1) above to make sense?
Note: you will need to be general here (for example "time") since explicit measurements (for example "seconds") were not given yet.

Problem 2. Find all values of the damping constant γ such that the system is underdamped. Recall that, in this case, the system will oscillate, but the amplitude will decrease over time.

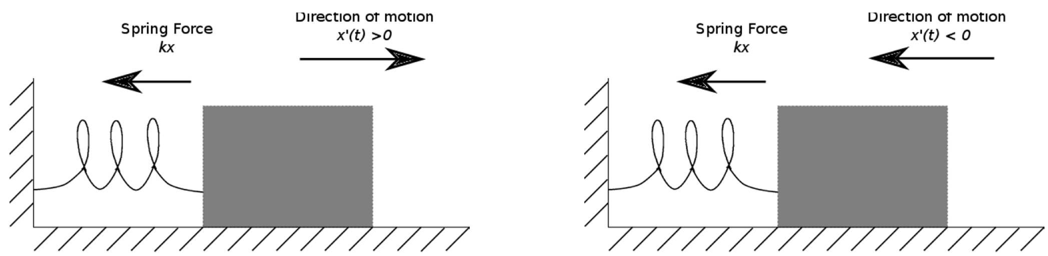
Problem 3. Show that the total energy of the system is not conserved in this case (of an underdamped system). What is happening? Justify your conclusion.

Part B – Coulomb Damping

Since most structures are underdamped, friction devices are frequently installed in buildings to minimize oscillations. **Coulomb damping** is a type of (constant) mechanical damping caused by the friction of one object sliding across another. This damping force opposes motion, but *is independent of velocity*. Consider the simple case of a block of wood sitting on a table and attached to a spring. In this system, the Coulomb damping force is the friction between the wood block and the table. See Figure 2 below.

Problem 4. Explain why the model in (1) is not valid in this situation.

Problem 5. Assume that the Coulomb damping is a constant $F_0 > 0$ (independent of velocity) that acts in the opposite direction of motion. Finish drawing arrows in the diagram in Figure 2 to



(a) Motion in the positive direction ($x'(t) > 0$) (b) Motion in the negative direction $x'(t) < 0$)

Figure 2: We need to add arrows to show which direction the Coulomb damping force is acting in each case.

represent the direction of the Coulomb damping force in each case. Does the Coulomb damping force (F_0) act in the same or opposite direction as the spring force (kx) in each case? Use this information to write an initial-value problem to model this situation (where the only damping is Coulomb damping), taking the direction of the damping into account. *You should find two differential equations here that depend on which way the motion is occurring.*

Problem 6. Assume that motion is in the positive direction (i.e. $x'(t) > 0$). Solve the initial-value problem from Problem 5.

Problem 7. Show that the total energy is not conserved in this case. What is happening? Justify your conclusion.

ACTIVITY 2 – VIBRATIONS

If an earthquake occurs, the ground will be displaced some distance $x_G(t)$. Then that total lateral deflection of the roof is

$$x_T(t) = x(t) + x_G(t)$$

where $x(t)$ is the displacement caused by vibrations of the building and $x_G(t)$ is the movement caused by the movement of the ground (and therefore, the supporting column).

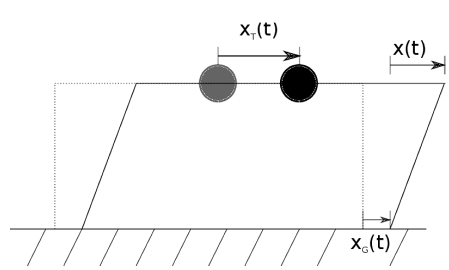


Figure 3: Displacement of a roof caused by an earthquake.

Suppose that your system is experiencing viscous damping. Then our model is

$$mx_T'' + \gamma x' + kx = 0, \qquad x(0) = 0,\ \ x'(0) = 0$$

since only the roof/building (not the ground) experiences the viscous damping. As in Part I [6], we will take

$$x_G''(t) = -\frac{A}{m}\sin(\omega t).$$

Problem 8. Assuming that the system is underdamped, find the general solution of $x(t)$; you do not need to find any coefficients. Explain why you know this is the correct solution form.

Problem 9. Assume that the roof of a small building has a mass of 8000 kg and the stiffness of the column is 9000 kg/s^2. The viscous damping built into the design is only 30 kg/s. You know that the roof (and building) will collapse if the roof moves more than one meter *from the base of its supporting column.* Since the column, roof, and ground move together from the ground vibration $x_G(t)$, this means that we need $x(t)$ (not $x_T(t)$) to be less than one meter to avoid disaster. If there is a vibration (say, take $A = 100$), what values of ω will cause the building to collapse? *Consider only the long-term vibrations that do not quickly dissipate.*

MAIN GOAL

Problem 10. Assume that the roof of a small building has a mass of 8000 kg and the stiffness of the column is 9000 kg/s^2. Assume that the largest earthquake (in recent history) in the area can be modeled by

$$x_G''(t) = -1.25\sin\left(\frac{\pi}{2}t\right).$$

What is the smallest value of the viscous damping coefficient γ necessary when designing the building to prevent the roof from moving more than 0.5 meters from the base of its supporting column? *Consider only the long-term vibrations that do not quickly dissipate.*

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