The route to chaos in a dripping water faucet

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that they did on this experiment last quarter. We would also like to thank Erik Swenson for his much appreciated help with graphing.

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The route to chaos in a dripping water faucet[a)]

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The dripping water faucet is a simple system which is shown in this article to be rich in examples of chaotic behavior. Data were taken for a wide range of drip rates for two different faucet nozzles and plotted as discrete time maps. Different routes to chaos, bifurcation and intermittency, are demonstrated for the different nozzles. Examples of period-1, -2, -3, and -4 attractors, as well as strange attractors, are presented and correlated to the formation of drops leaving the faucet.

I. INTRODUCTION

During the past decade, there has been increasing interest in dynamic systems which exhibit chaotic behavior. The term "chaotic" is generally used to describe nonlinear, but deterministic systems whose dynamic behavior proceeds from stable points through a series of stable cycles to a state where there is no discernible regularity or order. A permanent magnet in an oscillating magnetic field,[1] a bounding ball,[2,3] an inverted pendulum,[4] and a bipolar motor[5] represent some of the mechanical systems which exhibit this behavior.

In 1977, Rossler[6] suggested that a dripping water faucet might exhibit chaotic transition as the flow rate is varied. His prediction was experimentally confirmed a few years later by Shaw.[7] Since then, the nonlinear behavior of a dripping faucet during the transition to chaos has been examined by several authors.[8–12]

One way to study the dynamics of a dripping faucet is to measure the time interval t_N between successive drops. The time series thus obtained can then be plotted in a time-delay coordinate system (t_{N+1} vs t_N) to obtain a two-dimension-

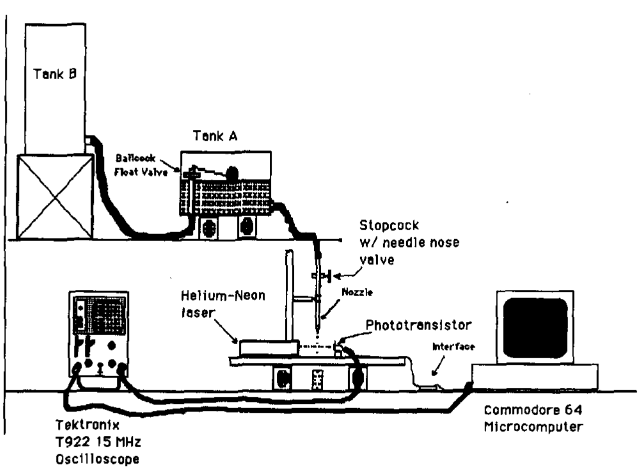

Fig. 1. Block diagram of apparatus used to measure drip rates.

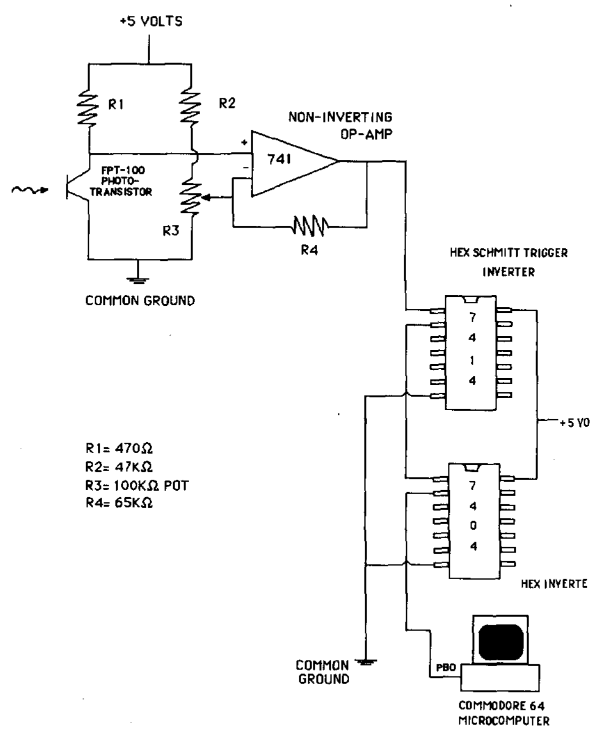

Fig. 2. Circuit used to amplify and shape signals generated by the phototransistor.

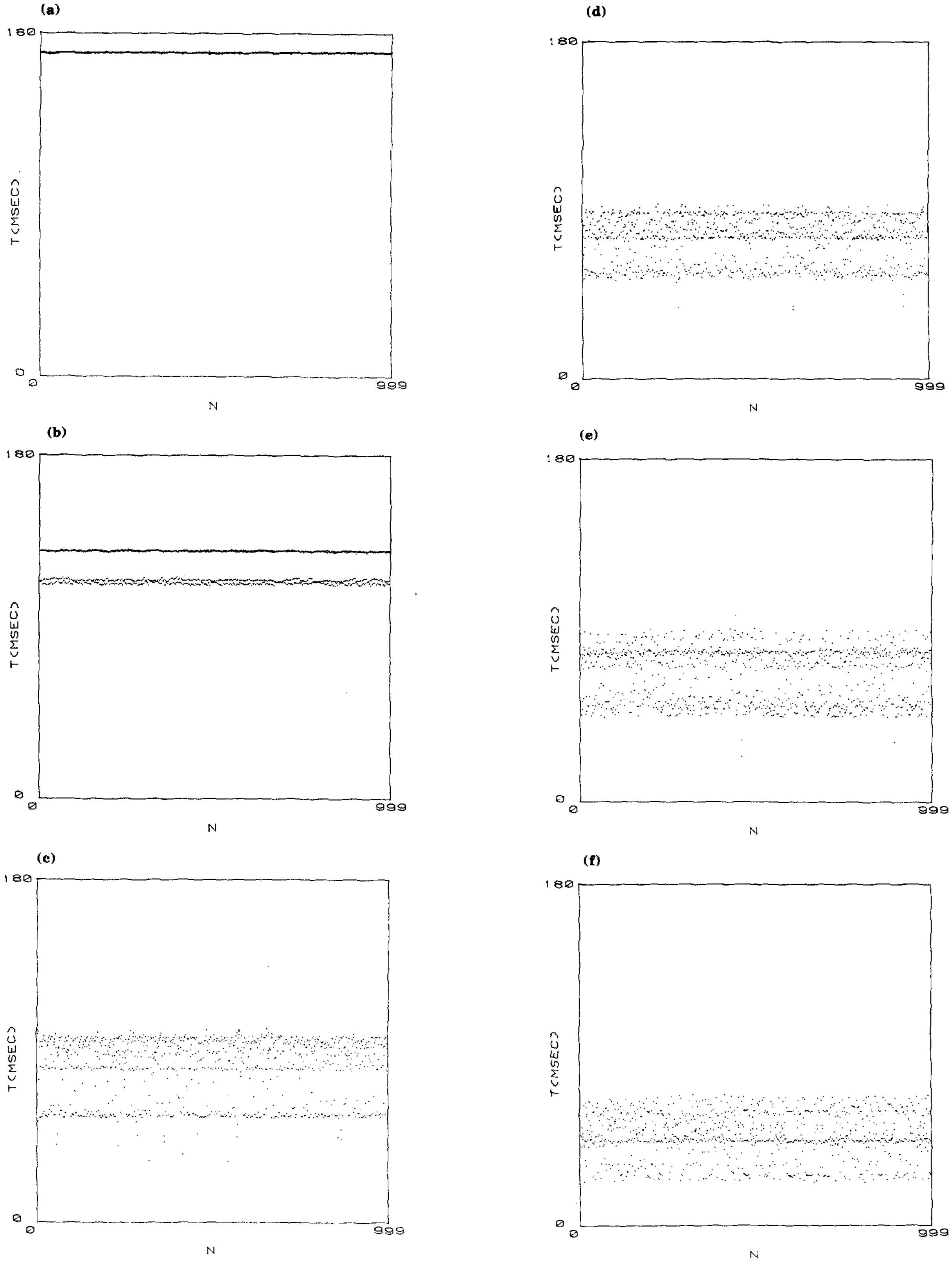

Fig. 3. Time interval between drops versus drop number for 1000 drops from the 1.95-mm nozzle. The time series here correspond to the discrete time maps shown in Fig. 9. (a) Period 1 at 5.9 drops/s, (b) bifurcation at 8.2 drops/s, and strange attractors at (c) 12.4, (d) 13.6, (e) 14.7, and (f) 21.0 drops/s.

al graph known as a discrete time map. A constant drip rate, when plotted in this way, will produce a single point known as a period-1 attractor. This corresponds to individual drops leaving the faucet at equal time intervals. Figure 3(a) and 9(a) show an example of a period-1 attractor.

It is expected that as each drop leaves the faucet, it creates oscillations in the residue at the nozzle which set the initial conditions for the following drop, so that the successive drops are casually related. As the flow of fluid through the faucet is increased, the system proceeds

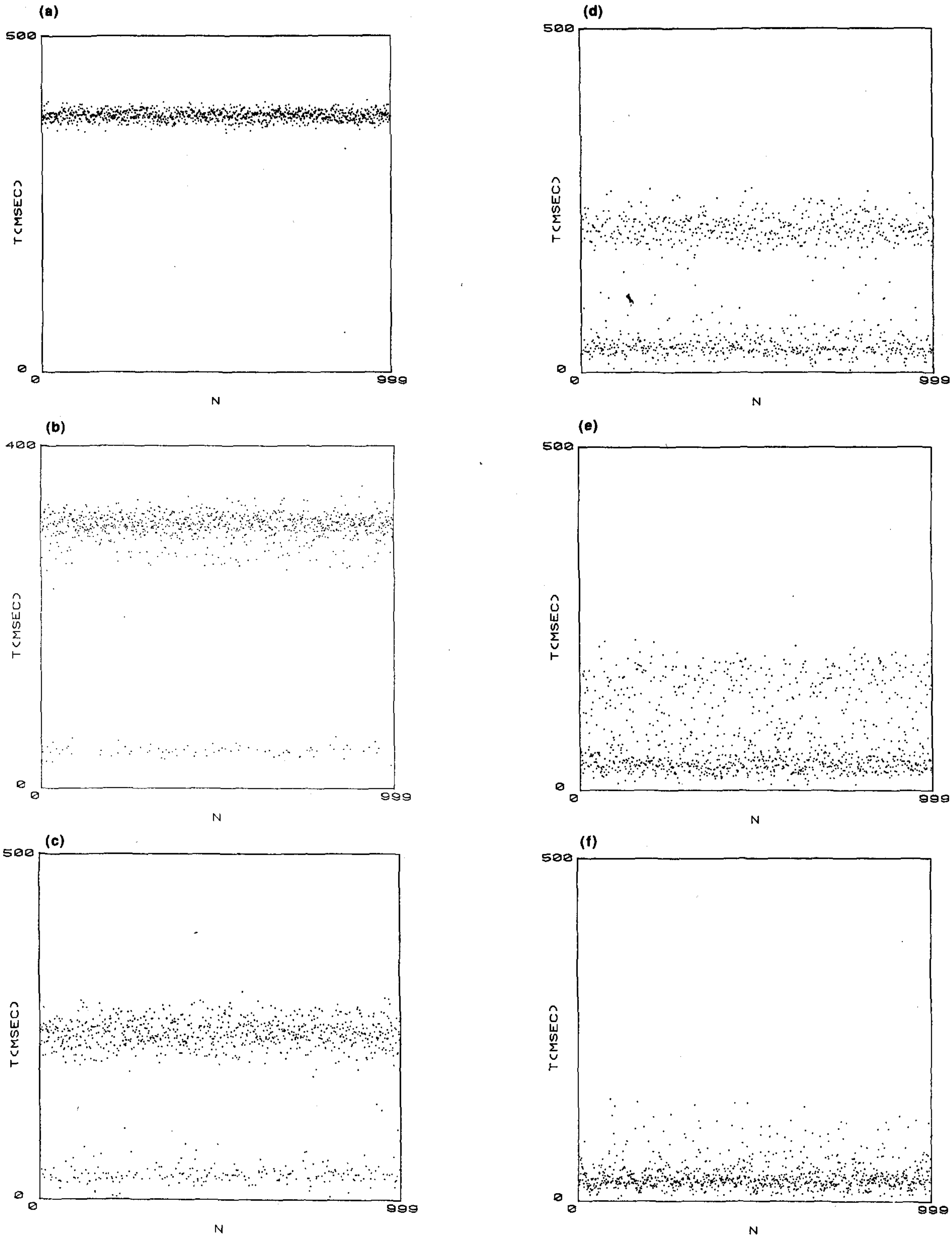

Fig. 4. Time interval in milliseconds versus drop number for 1000 drops from the 0.47-mm nozzle. The time series here correspond to the time maps shown in Fig. 10. (a) Period 1 at 2.6 drops/s, (b) period 3 at 3.5 drops/s, period 4 at (c) 5.1, (d) 8.1, and (e) 12.8 drops/s, and an L-shaped attractor at (f) 26.8 drops/s.

through a sequence of attractors until the flow becomes essentially continuous. One possible sequence is for the system to move from a constant drip rate to a pairing of drops leaving the faucet. This is an example of "bifurcation" and results in a period-2 attractor. Figure 3(b) provides an example of a time series showing bifurcation, and Fig. 9(b) is the corresponding time map. As the flow rate is increased further, period-4 attractors and more complicated "strange" attractors may occur.

This seems to be the primary route to chaos which has been previously reported for a dripping faucet, but it is not the only possible one.[13,14] If the drop rate is regular and stable at one flow rate, but is interrupted by "bursts" of other drops when the flow is increased slightly, the phenomenon is known as "intermittency." In this case, the "extra" drops will occur more frequently as the flow continues to increase. On a time map, the system will appear to develop a period-3 attractor rather than one of period 2.

In this article, we present two routes to chaos observed as the drip rate was increased for nozzles of different inner diameter. One of the nozzles used in this investigation had an inner diameter which is 50% smaller than any previously reported. This nozzle did not exhibit the standard bifurcation route to chaos, but rather it followed an intermittent route involving period-3 and -4 attractors. The larger nozzle exhibited bifurcation for low drip rates and strange attractors at high drip rates with a small range of instability between the two.

To relate specific drip patterns to points on the corresponding discrete time maps for each nozzle, a videotape of the formation of drops was made. This videotape was also used to help ensure that no drops were being missed by the apparatus.

II. APPARATUS

Figure 1 shows a sketch of the apparatus used to measure the time interval between successive drops of distilled water. A large reservoir in tank B was connected to tank A with a ballcock float valve which serves to regulate the water level in tank A. With this design, the pressure fluctuation at the nozzle never exceeded $\pm 0.25\%$ for any given trial. The water flowed from tank A through a rubber tube to a glass burette and exited from a glass nozzle. The drops from this nozzle passed through a He–Ne laser beam which illuminates a phototransistor.[15] The drip rate from the nozzle was controlled by opening a needle-nose valve in a stopcock in the burette.

A Commodore 64 computer was linked via an interface circuit to the phototransistor. When the laser beam illuminating this transistor was interrupted by a drop of water, an electrical impulse was sent from the phototransistor to the interface circuit shown in Fig. 2. The op–amp in this circuit amplified the signals from the phototransistor to ensure that they were all above the positive-going threshold voltage of 1.74 needed for the 7414 chip. This Schmitt trigger chip then produced a 5-V signal that was inverted and sent to the computer where a machine language program recorded the time measured on the computer's internal clock.

The machine language program has a resolution of approximately 55 μs. When tested with a Hewlett-Packard 331a function generator, the subroutine produced accurate results for frequencies up to 10 kHz, which corresponds to a drip rate of 10 000 drops/s. Since drip rates in this investigation never exceeded 30 drips/s, this is well above the drip-rate resolution needed.

Two different glass nozzles were used in this investigation. The first has an inner diameter of 0.47 ± 0.1 mm and an outer diameter of 3.06 ± 0.01 mm and is tapered at the end. The second nozzle, which has an untapered cylindrical shape, has an inner and outer diameter of 1.95 ± 0.01 and 3.15 ± 0.01 mm, respectively.

A Tektronix T922 15-MHz oscilloscope was used to monitor simultaneously the signal coming from the phototransistor as well as the signal entering the computer to ensure that all drops which passed through the laser beam were counted by the computer. It is possible, however, that some drops do not fall vertically downward and therefore do not interrupt the beam. To examine drop formation at the nozzle in detail, as well as to ensure that the detector was counting every drop, the flow from the faucet was videotaped using a high-speed video camera. The shutter of

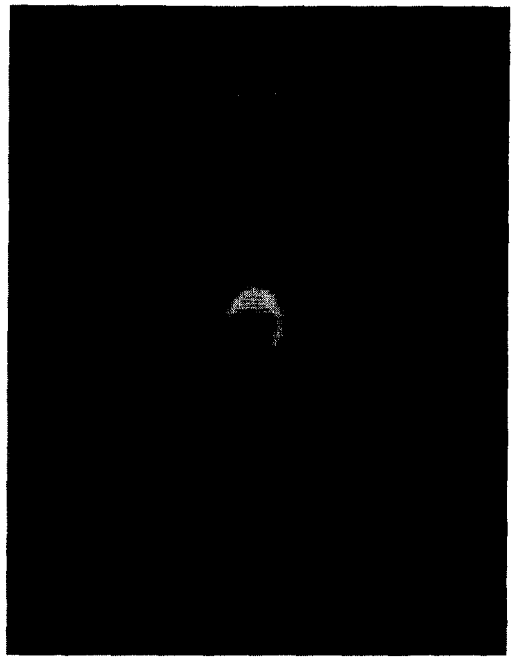

Fig. 5. Photograph showing a large drop followed by a tiny drop leaving the 1.95-mm nozzle. The tiny drop is always produced at low drop rates for this nozzle, but they do not always pass through the laser beam and are not always detected by the apparatus.

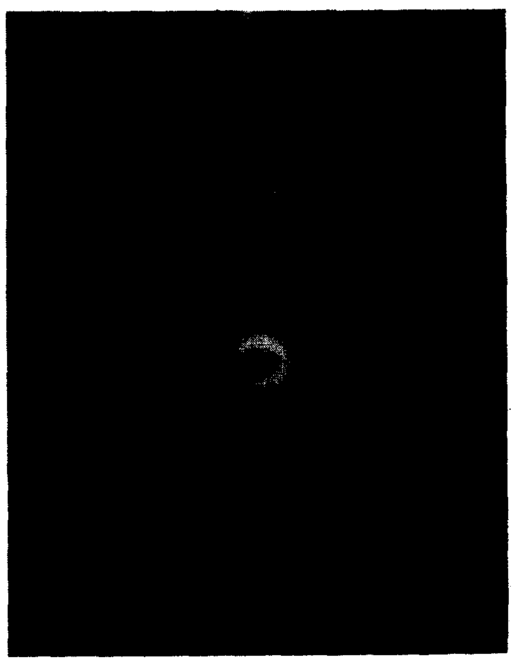

Fig. 6. Photograph showing a pair of drops leaving the 0.47-mm nozzle at 3.5 drops/s. Only single drops are produced until a rate of 2.6 drops/s when these secondary drops begin to appear very occasionally. As the drip rate is increased, the secondary drops appear with increasing frequency. The small drops shown in this photograph are easily detected by the apparatus.

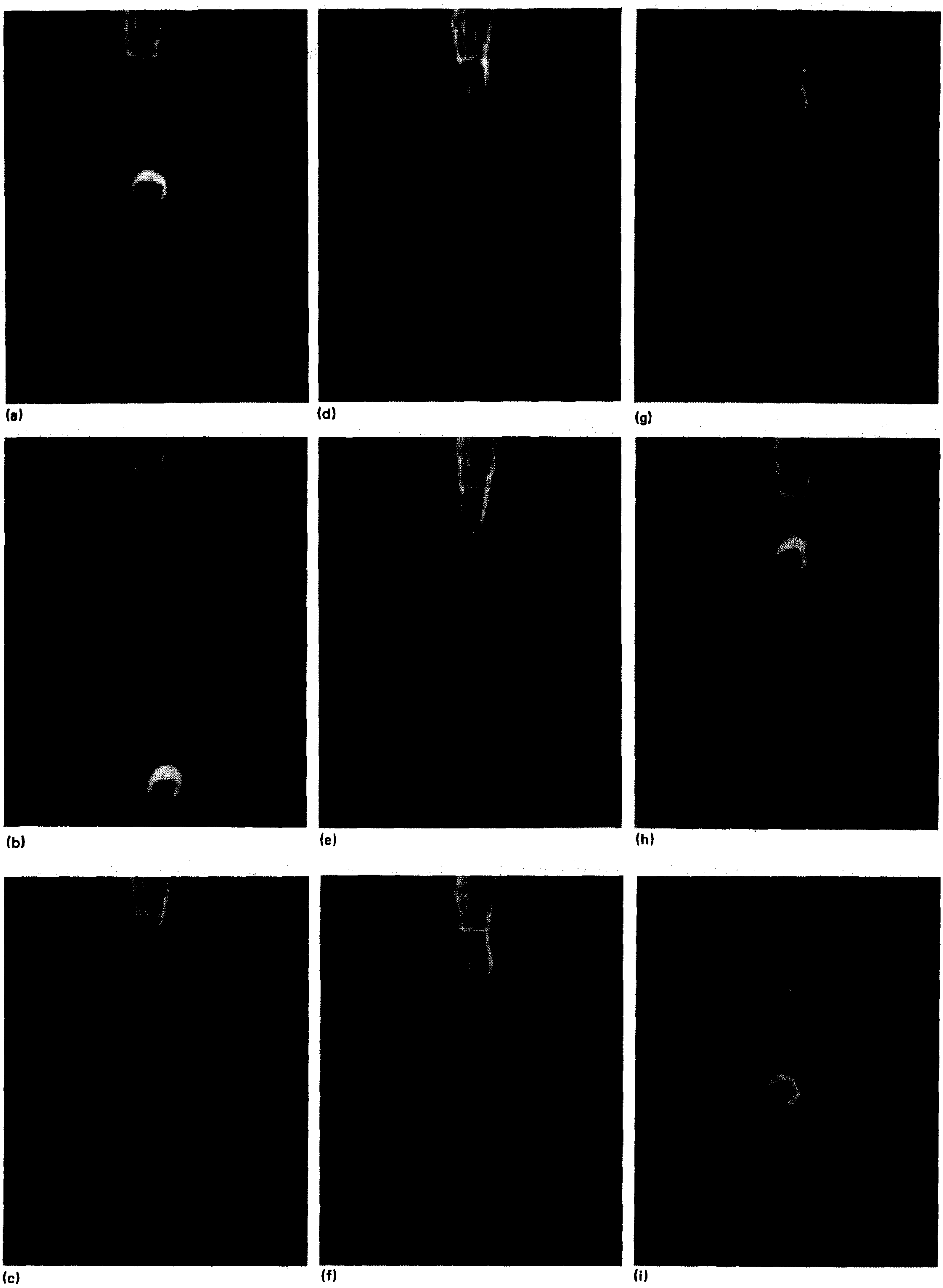

Fig. 7. Sequence of drops formed at 3.5 drops/s for the 0.47-mm nozzle. Each exposure corresponds to $\frac{1}{1000}$ s and the time between frames is $\frac{1}{30}$ s. When the large drop breaks off, the residue at the nozzle oscillates in three dimensions. Finally, a large-drop/small-drop pair separates from the nozzle. The time map for this drop sequence is shown in Fig. 10(b).

this camera exposes a charge-coupled device (CCD) for 1/1000th of a second, and then the CCD is scanned to record an image on videotape. This process is repeated 30 times a second, resulting in a sequence of 30 pictures/s, each of which was exposed for 1/1000th of a second.

Cahalan *et al.*[12] have shown that some errors which can occur in a dripping faucet experiment are evident if the data are plotted as a time series (time interval versus drip rate). Time series for the data taken for this study have been plotted and are shown in Figs. 3 and 4. Any change in drip rate caused by pressure variations at the nozzle or changes in environmental conditions should appear in these plots as a systematic drift toward shorter or longer time intervals. No such drift is evident in the data.

III. DESCRIPTION OF DROPS

At low drip rates, the larger of the two nozzles used in this investigation (1.95 mm i.d.) always produces a large drop, approximately 4.5 mm in diameter, followed by a tiny drop whose diameter is less than 20% that of the large drops (Fig. 5). Some of these tiny drops leave the nozzle at an angle, do not pass through the laser beam, and are not counted by the detector.

This behavior continues for drip rates up to about 9.0 drop/s. At this rate, the tiny drops disappear quite abruptly and only single large drops are produced. This causes the measured drip rate to decrease to about 5.0 drops/s even though the valve opening allows more water to flow through the nozzle. The flow stabilizes quickly, and a sequence of large drops leaves the nozzle by about 5.5 drops/s. As the flow rate is increased, the drop size varies slightly and the time between drops varies in a nonlinear manner. For high drip rates (of the order of 20 drops/s) drops sometimes appear to leave the faucet in pulses of several drops.

Flow from the smaller faucet (0.47 mm i.d.) behaves somewhat differently. The tiny drops described above are produced only for drip rates below 1.0 drops/s. For drip rates between 1.0 and 2.6 drops/s, only large drops are produced. At 2.6 drops/s, small drops occur *very occasionally* between the large drops, but these small drops are much larger than the tiny drops produced at very low drip rates, measuring between one-third and one-half the diameter of the larger drops (Fig. 6). These smaller drops fall essentially straight down and are always detected by the timing apparatus. Figure 7 shows a sequence of these drops forming. The large lateral motion of the residue at the nozzle shown in this sequence was apparent only for this nozzle and only for drip rates between about 2.5 and 4 drops/s. This may be related to the relatively small inner diameter or relatively thick glass walls of the smaller nozzle used, or it may be due to some other geometric factor such as a tiny asymmetry in the nozzle.

As the flow rate is increased further, these smaller drops are produced more frequently. At about 3.9 drops/s, the flow becomes more complicated with a mixture of smaller and larger drops leaving the nozzle in spurts (Fig. 8). For this and higher drip rates, each large drop is followed by zero, one, two, or three smaller drops. Furthermore, drop size varies from drop to drop. Beyond about 11 drops/s, the spacing between drops appears random to the eye.

IV. TIME MAPPINGS

The time mappings show different routes to chaos for each of the two nozzles used. As described in Sec. III, low drip rates from the larger of the two nozzles (1.95 mm i.d.) produces very tiny drops which are not always detected by the apparatus. These drops escape detection because oscillations at the nozzle cause the smaller drops to be released at an angle and they do not pass through the path of the laser beam. By visually examining the data, one can pick out these missed drops. The data also suggest that every tiny drop is paired with a larger drop, and a discrete time map would show bifurcation. Further examination of these drip rates with videotape seems to support this conclusion.

When the drip rate has increased so that the tiny drops are no longer produced, the large drops leave the nozzle at a constant rate, resulting in a period-1 attractor [Fig. 9(a)]. This occurs at a drip rate of approximately 5.5 drops/s and continues until the rate reaches 7.0 drops/s. For drop rates between 7.0 and 7.5 drops/s, the system is unstable, producing both period-1 and -2 attractors. From 7.5 to 9.0 drops/s, primarily period-2 attractors were seen [Fig. 9(b)]. This is the bifurcation route to chaos reported by previous investigators.

Between about 9.0 and 10.5 drops/s, the system settles into a constant drip rate, and period-1 attractors once again dominate. However, the system is somewhat unstable in this range, and occasionally period-2 attractors or even strange attractors appear in the data. For drip rates higher than 10.5 drops/s, strange attractors are produced [Figs. 9(c)–9(f)]. The strange attractor in Fig. 9(d) shows a box formation suggestive of a period-4 attractor. The other attractors are similar to those previously reported in the literature.

The results for the smaller nozzle (0.47 mm i.d.) show

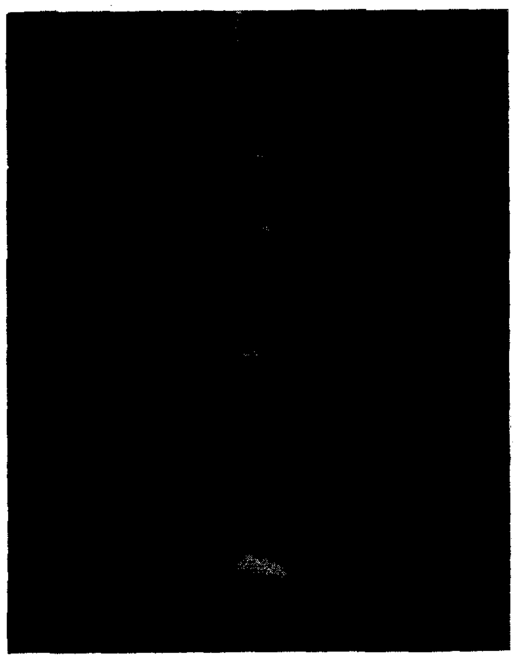

Fig. 8. At 6.8 drops/s, four drops leave the 0.47-mm nozzle in sequence. For drip rates in this range, each large drop may be followed by zero, one, two, or three smaller drops in a sequence like the one shown.

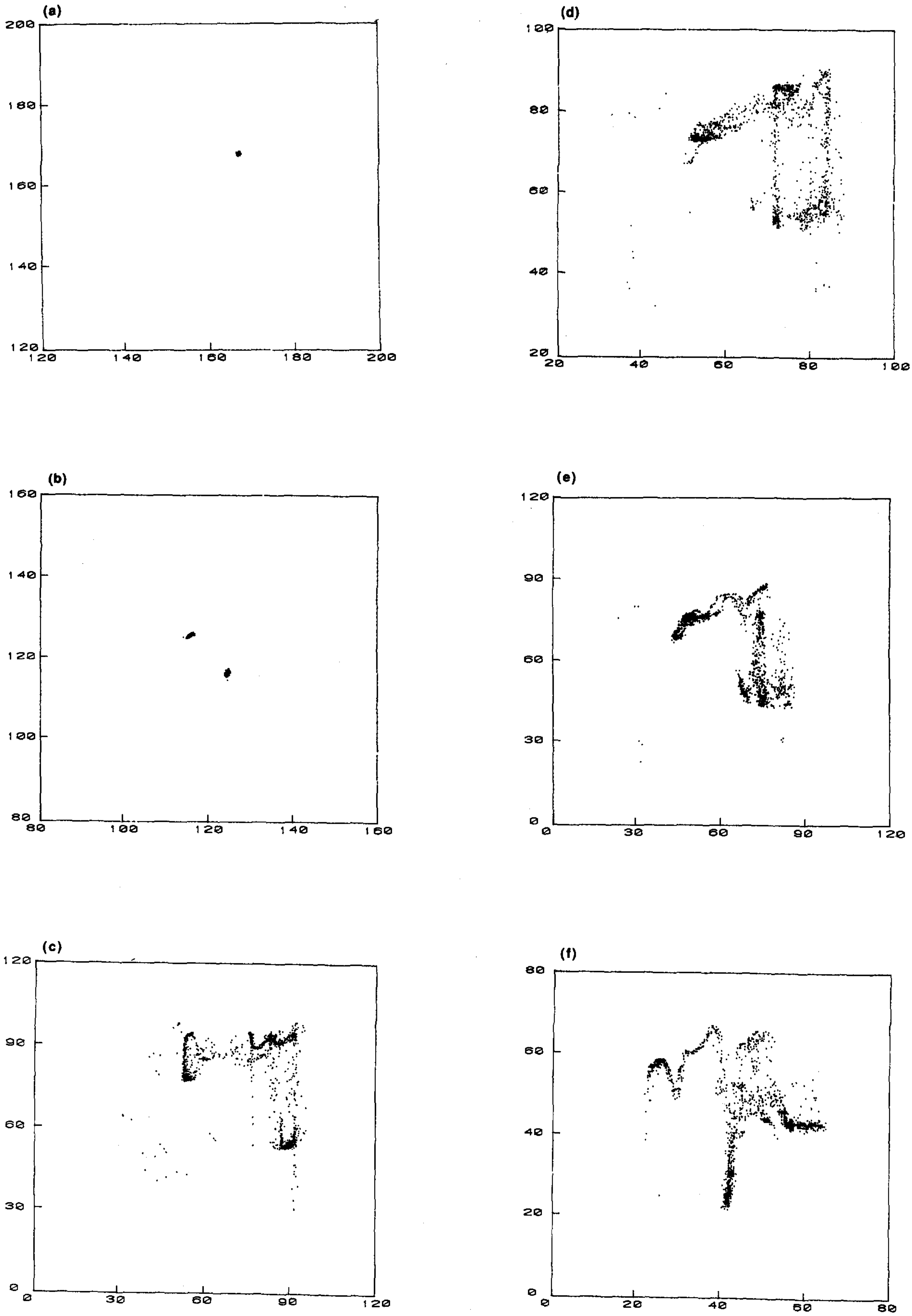

Fig. 9. Examples of t_{N+1}-vs-t_N time maps plotted for the 1.95-mm nozzle. (a) At 5.9 drops/s, a period-1 attractor results. (b) By 8.2 drops/s, this has bifurcated to produce a period-2 attractor. (c)–(f) Strange attractors dominate for drip rates above 10.5 drops/s. The attractors shown here correspond to drip rates of 12.4, 13.6, 14.7, and 21.0 drops/s, respectively. All times are measured in milliseconds.

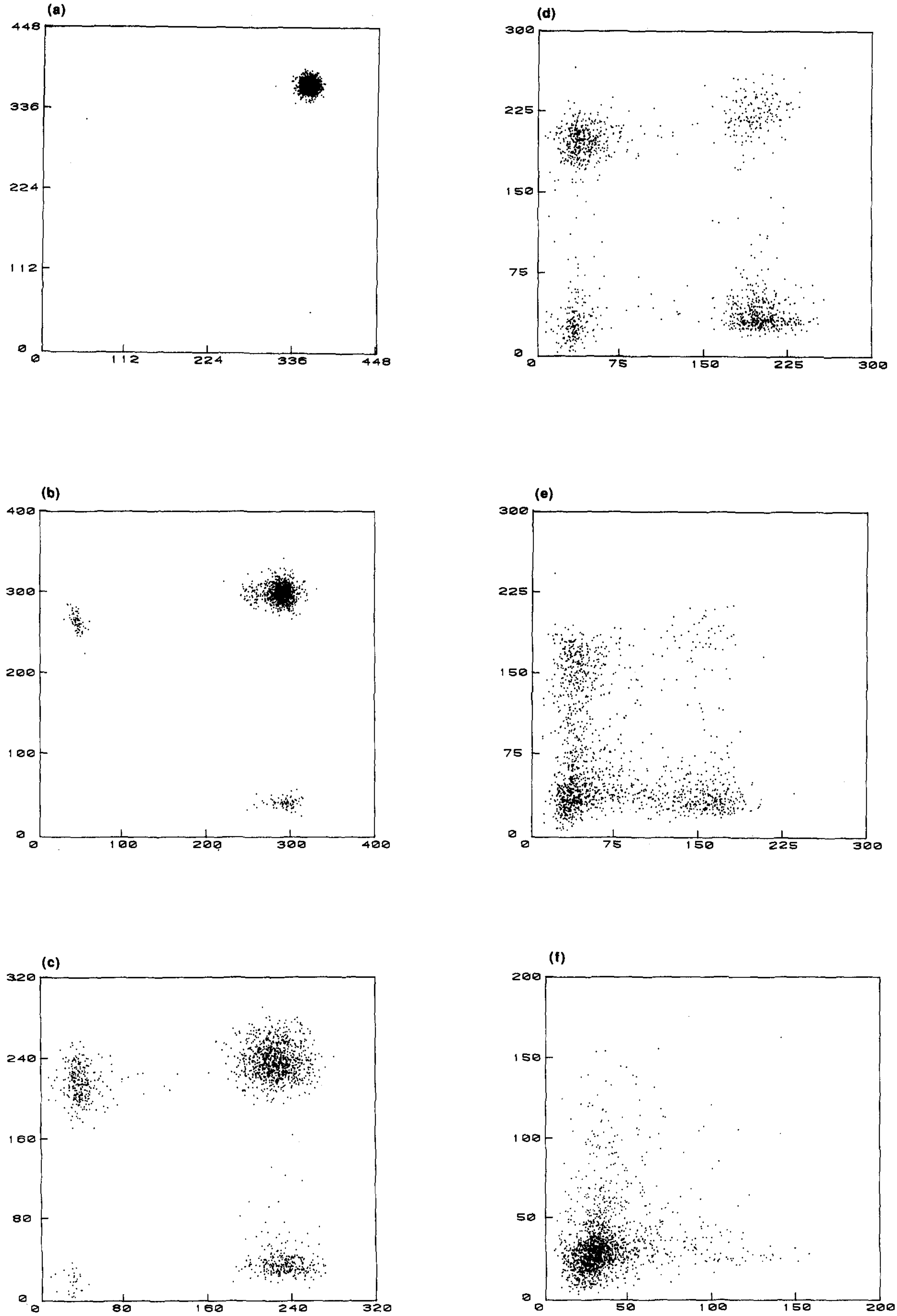

Fig. 10. Examples of t_{N+1}-vs-t_N time maps plotted for the 0.47-mm nozzle. (a) At 2.6 drops/s, the period-1 attractor shows the first indication of a developing period-3 attractor. (b) By 3.5 drops/s, the period-3 attractor has fully developed. (c) A period-4 attractor develops by 5.1 drops/s, (d) is fully developed by 8.1 drops/s, and (e) begins to disappear by 12.8 drops/s. (f) By 26.8 drops/s, an L-shaped attractor results as most points cluster in the lower left corner. All times are in milliseconds.

an intermittent route to chaos. For drip rates between 1.0 and 2.6 drops/s, only a single large drops are produced and the time interval between successive drops is constant. A time map in this region produces the period-1 attractor shown in Fig. 10(a).

At 2.6 drops/s, smaller drops occur between the large drops very occasionally. A time map in this region shows the beginnings of a period-3 attractor. Most of the data follow the map for a period-1 attractor, but the occasional pairing of a large drop/smaller drop produces the two extra point groupings in the upper left and lower right corners of this map. By 3.5 drops/s [Fig. 10(b)], the period-3 attractor has fully developed.

As the drip rate continues to increase, more than one small drop appears between the large drops, and by 5.1 drops/s, a period-4 attractor has formed, shown in Fig. 10(c). The upper right corner of Fig. 10(c) corresponds to two or more large drops leaving the nozzle in sequence. The time interval between small drops is less than that between large drops, and so two or three small drops in sequence produce the grouping at the lower left corner of the mapping. the upper left and lower right corners correspond to either a big drop followed by a small drop or a small drop followed by a big drop, respectively.

The period-4 attractor begins to change to an L-shaped attractor by 12.8 drops/s [Fig. 10(e)] as single large drops become rarer. Eventually, the time between drops becomes very small and the points on the time map migrate toward the lower left corner [Fig. 10(f)].

V. SUMMARY

The dripping water faucet is a simple system which has been shown in this article to be rich in examples of chaotic behavior. Two different size and shape faucet nozzles were used to measure the time interval between drops for a wide range of drip rates. Flow of water from the nozzle was videotaped, and groupings of points on the discrete time maps were correlated with drop formation at the nozzle. This proved to be a very useful tool in understanding drip patterns, and its use in future investigations is recommended.

Drops from the larger of the nozzles used in this study were seen to follow a bifurcation route to chaos producing period-1 and -2 attractors at lower drip rates and many beautiful examples of strange attractors for higher drip rates with a range of instability between the two regions. Data taken for the smaller nozzle exhibited an example of an intermittent route to chaos, with period-3 and -4 attractors. The different routes to chaos may be related to the relatively small inner diameter or relatively thick glass walls of the smaller nozzle used, or it may be due to some other geometric factor such as a tiny asymmetry in the nozzle. In any case, the results observed in this study suggest that further examination of drip rates for a wide range nozzle sizes and geometry should be undertaken. The apparatus needed is inexpensive and simple to duplicate and nozzles can be readily constructed from glass tubing.

ACKNOWLEDGMENTS

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[a] This paper is based on a thesis of K. Dreyer while a senior at Hartwick College.

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[15] FPT-100 phototransistors were used in this experiment because they were easily available within the department. They may no longer be available commercially, but any phototransistor with a rise time of a few microseconds should be satisfactory.