

STUDENT VERSION

Analyzing the Spring-Mass Model Against Empirical Data in a Lab

Bonnie Moon, Department of Mathematics
Brigham Young University-Idaho, Rexburg ID USA
moonb@byui.edu

Abstract: In this lab students will collect data on their spring mass systems and compare their empirical models to their theoretical ones—giving them an opportunity to actually test a model against data. Before this lab, students should have modeled spring-mass systems and solved second-order linear differential equations with constant coefficients. The lab will take one 50-minute class period with some outside student preparation to be finished before lab day.

Keywords: mass-spring system, lab, data, parameter estimation, model validation

Tags: second order, linear, differential equation, theoretical model, empirical model

Preparation before Lab Day

At least two Members of each group need to download the Logger Pro software onto their laptops and be sure to bring their laptops to class on lab day. If your school does not have Logger Pro or the equipment needed to conduct the experiment, there are videos available where this experiment was recorded.

Part I: Before Lab Day--Theoretical Modeling

1. Write down the second order differential equation associated with the mass-spring system using m to stand for mass in kilograms, k for Hooke's constant in N/m, and c for the damping coefficient in kg/sec. Assume there is no forcing function acting on the system.
2. In a short paragraph explain the spring-mass system. Here include specifically what each variable represents and its units in the above equation.
3. Do some exploring (on the internet is fine) and find out where a mass spring system could be applied. Briefly describe your example in two-three sentences.
4. Solve the differential equation in #1 above when the damping coefficient is zero. Show your work and state the solution in terms of m and k .

Part II: Lab Day—Empirical Modeling--Collect the Data

- 1. Review Part I before conducting the experiment.** Before lab starts, check your answers to Part I with your group. Check your spring-mass model and solution with your other group members. Share some of the applications you found for spring-mass models.
- 2. Set up the Experiment (This set-up should have been completed by a few members of your group before class started.)** Set up your experiment station with your group. Each station needs a laptop with Logger Pro downloaded and an ultrasonic sensor connected to the laptop with Logger Pro.
- 3. Collect the Data** Have one member of your group obtain a mass and spring. This mass and spring will need to hang freely from a rod—like a pencil set up between two desks. Your sensor will need to be close enough to the mass and below the mass so that it will read the oscillations. You may need to place your sensor on a box or on books.

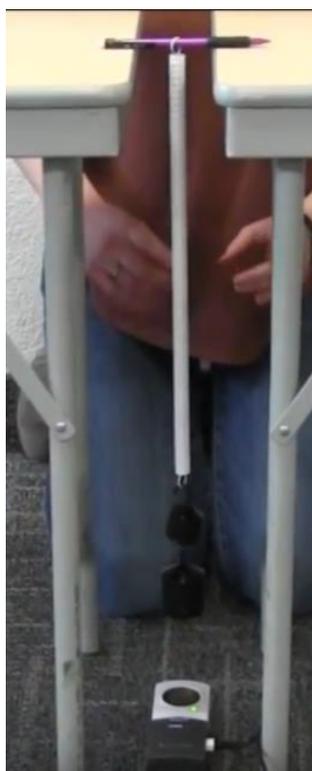


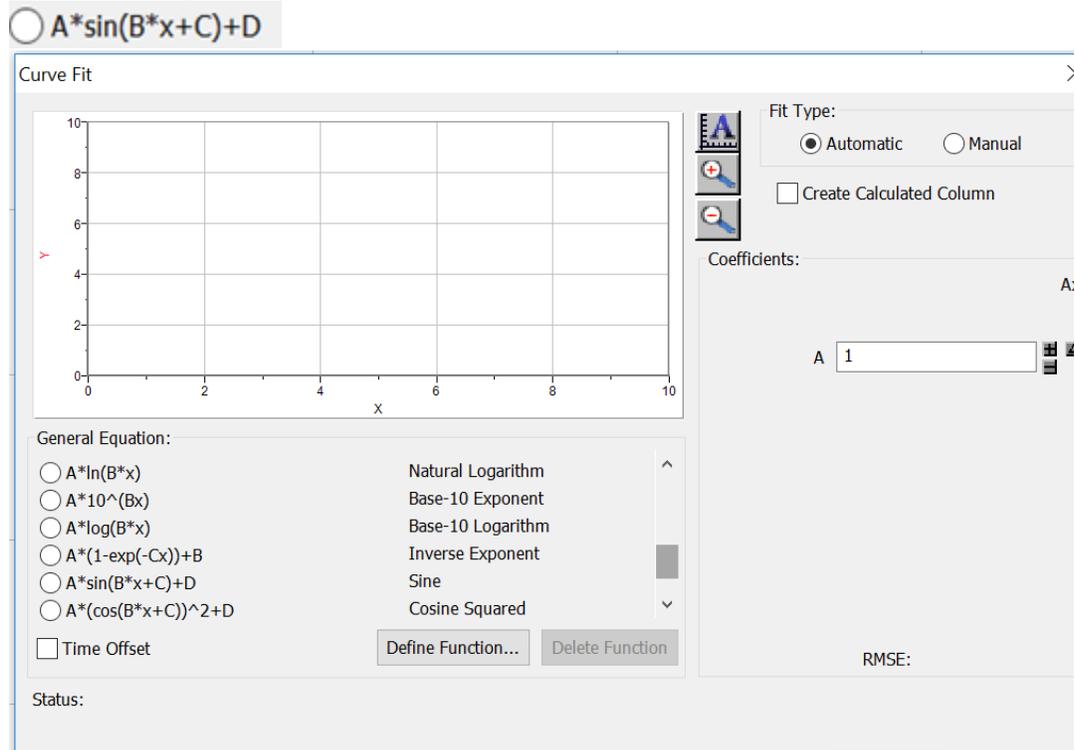
Figure 2. Experiment Set Up

With your group you will collect data for two different mass/spring systems. For each system, conduct the experiment three times and write down the coefficients A, B, C and D produced by the software for the curve:

$y_{fit} = A \sin(Bt + C) + D$. Parts a, b and c below explain how to do this in more detail.

- Start your spring-mass system in motion by displacing the mass below the equilibrium point. As soon as you let go of the mass, make sure your Logger Pro group member hits Start on the computer. You should see Logger Pro collecting and recording data in the left two columns of the screen. At the end of about 30 seconds, click “STOP.”

- b. Now your Logger Pro group member will go to the upper menu bar and select “Curve Fit.” In the pop-up menu scroll down until you see the sin curve. Select this curve for your curve fit.



After you hit “Curve Fit”, four values will show up for A, B, C and D respectively. Record these values in the chart below.

- c. You will complete three trials of this experiment with the same mass-spring system and continue to record your data in the chart below. When you complete this, your group will then select a different mass and spring pair and complete three trials of the experiment again.

4. **Record the Data** Fill in the chart below with the information obtained from Logger Pro. After recording your coefficients, compute and record the averages of the three trials for each coefficient as well.

Experiment 1

Mass =

Spring Constant =

	A	B	C	D
Trial 1				
Trial 2				
Trial 3				
Average of 3 Trials				

Experiment 2

Mass =

Spring Constant =

	A	B	C	D
Trial 1				
Trial 2				
Trial 3				
Average of 3 Trials				

Part III: Compare your Theoretical Results with Your Empirical Results.

1. Calculate the theoretical solutions for your two spring-mass systems and state these solutions here. You did this general solving already in Part I. Now just substitute your m (mass) and k (Hooke's constant) in the solved equation.

Experiment 1 Theoretical Solution:

Experiment 2 Theoretical Solution:

2. Write out the empirical solutions you obtained in your two experiments. Use the average values for each of your constants A, B C and D for each solution.

Experiment 1 Empirical Solution:

Experiment 2 Empirical Solution:

3. Let's compare your results. You may notice here that your empirical solutions are in terms of just the sine function while your theoretical solutions are in terms of the cosine and sine functions. Using trigonometry, you can actually transform the empirical solutions you obtained to match the form of your theoretical solutions. For the purpose of this exercise, we will skip the details of this mathematics and focus on the periods of our two solutions for comparison. We can compare the period of the theoretical model directly to the period of the empirical model without any need to transform the solutions into the same form. Fill in the chart below with the periods of your two theoretical solutions alongside the periods of your two empirical solutions. (Round to the nearest thousandth.)

	Experiment #1	Experiment #2
Period of theoretical solution ($B * 2\pi$)		
Period of empirical solution ($\sqrt{k/m} * 2\pi$)		

In two to three sentences describe how the periods compare between your theoretical solutions and empirical solutions. Explain why you are possibly seeing what you do in both scenarios. Is one or both of your empirical periods significantly different than their respective theoretical ones? Give possible reasons that your periods may differ.

4. Do you think your theoretical model and solution do a good job predicting the behavior of your spring-mass system? Explain. What more would you like to know to better compare your theoretical solutions against your empirical ones?