# STUDENT VERSION
# FALLING COFFEE FILTERS

## STATEMENT


**Figure 1.** Vernier Software & Technology Motion Detector and stack of coffee filters.

We are given data on time and position of a free falling stack of coffee filters. The data offered in Table 1 is from an experiment to model a falling stack of 12 commercial coffee filters. Each coffee filter has mass 0.01908 kg for total stack mass $m = 12 \times 0.01908 = 0.22896$ kg. The data was collected with a Vernier Software & Technology Motion Detector on 20 February 2010 at the United States Air Force Academy CO where the acceleration due to gravity is $g = 9.78976 m/s^2$. The Motion Detector, along with a stack of coffee filters, is depicted in Figure 1 while a plot of the data from Table 1 is offered in Figure 2.

We will make a mathematical model of this phenomenon and estimate the parameters in the model. We shall also compare the model's predictive power with the actual data.

| Time (s) | Distance (m) | Time (s) | Distance (m) |
|---|---|---|---|
| 0.00 | 0.0000 | 0.22 | 0.3619 |
| 0.02 | 0.0228 | 0.24 | 0.4050 |
| 0.04 | 0.0478 | 0.26 | 0.4500 |
| 0.06 | 0.0730 | 0.28 | 0.4931 |
| 0.08 | 0.1054 | 0.30 | 0.5440 |
| 0.10 | 0.1369 | 0.32 | 0.5927 |
| 0.12 | 0.1710 | 0.34 | 0.6429 |
| 0.14 | 0.2063 | 0.36 | 0.6937 |
| 0.16 | 0.2426 | 0.38 | 0.7458 |
| 0.18 | 0.2810 | 0.40 | 0.7991 |
| 0.20 | 0.3197 | 0.42 | 0.8556 |

**Table 1.** Data on the time and position of a stack of coffee filters as it falls to the ground. We measure the distance falling positively downward and say $y(t)$ is the distance fallen in meters at time $t$ in seconds, i.e. $t$ is time in seconds from release and $y(t)$. is distance in meters fallen from release.

We have actually taken our observed data and adapted it as follows to produce the data in Table 1. In order to estimate the initial velocity (for the equipment only offers position with averaging for its internal estimate of velocity) we use the first and third data point $(t_1, y_1)$ and $(t_3, y_3)$ where $t_i$ is time in seconds of the $i^{\text{th}}$ observation and $y_i$ is the distance fallen at the $i^{\text{th}}$ observation and form the symmetric difference about the second observation of our collected data as an approximation of initial velocity, calling this variable, AveVelInitial:

$$\text{AveVelInitial} = \frac{y_3 - y_1}{t_3 - t_1}\,.$$

In our case this was AveVelInitial $= 1.10446$ m/s. We then start our time at $t = 0$ at the second observation of our collected data and note our distance fallen here is 0 m, shifting all other time and distance observations accordingly, with an initial velocity of 1.10446 m/s.

a) Write down some assumptions about a controlled environment in which you might collect such data.

b) With these assumptions, construct a Free Body Diagram (FBD) and indicate all the forces acting on the mass of the stacked coffee filters.

c) Using Newton' s Second Law of Motion which says that the change in momentum is equal to the sum of all the external forces acting on the mass of the stacked coffee filters, construct a mathematical model for determining the velocity, $v(t) = y'(t)$, of this object. Here $y(t)$ is the distance the coffee filters have fallen from rest in meters at time $t$ in seconds.

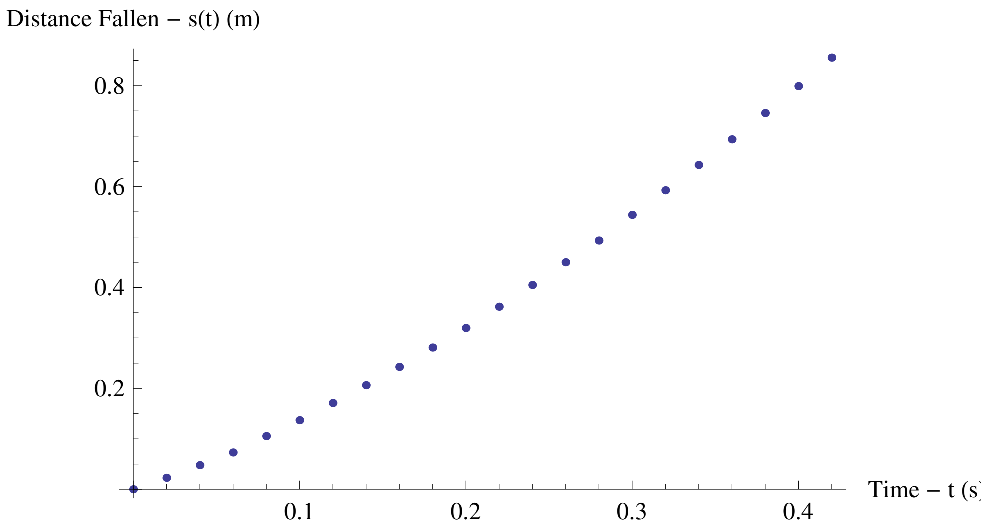


**Figure 2.** A plot of the data from Table 1.

$$d/dt(m * v(t)) = my''(t) = \underline{\hspace{6cm}}.$$

d) Solve your differential equation for $v(t)$ or $y(t)$.

e) If you choose to solve the equation in $v(t)$, the velocity, then you can integrate the velocity function you obtained, $v(t)$, above to get position function,

$$y(t) = \int_0^t v(T)\, dT\,.$$

Or you may solve the differential equation in (c) for $y(t)$, the position, and use that solution.

f) Estimate the parameters in your model.

g) Compare your solution of your model to the data offered above.

h) Write a paragraph description of the entire process in (a) - (g) and indicate how confident you are of the various aspects of this process.

i) What other variations might you consider for this model?